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ANALYSIS OF ACADEMIC PERFORMANCE IN THE DIFFERENTIAL CALCULUS COURSE IN THE MATHEMATICS PROGRAM OF THE UNIVERSITY OF CARTAGENA

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Abstract
Historically, teaching and learning mathematics area has been a difficult but important subject within the academic field. For this reason, it might have been used as a filter to have access to higher education. With the development of new education systems, countries face the challenge to improving math curriculum structure to offer students better opportunities to achieve the learning goals. A good example of this has been remedial mathematics. This study characterizes and analyzes, from a statistical point of view, math major students´ academic performance in their first and second semester at the University of Cartagena. They studied the Differential Calculus under different curricular reforms the Major has had (Presence or absence of a course prior to Differential Calculus). The data were statistically analyzed using the Mann-Whitney-Wicolxon and Kruskal-Wallis-Dunn's tests with Bonferroni adjustments. The results showed that students who took a remedial math course prior to the Differential calculous course got better academic performance than those who did not.

Key words: desertion, reform of the educational program, academic performance, calculation, probability. (Source: Unesco Thesaurus).

1. Introduction

Many everyday situations require mathematical thinking, since they help to form citizens with critical thoughts and increase the ability to reflect, solve problems and argue. (Quiroga, Coronado, & Quintana, 2011). Performance in the area of mathematics shown on tests such as Saber 11 (Colombian state proof), PISA, and public college admissions tests (Duque & Ortiz, 2013) In the particular case of the University of Cartagena they are worrisome, evidencing a deficiency in cognitive skills in this basic area which are necessary in higher education. (ICFES, 2011) (Universidad de Cartagena, 2017). The Ministry of National Education states that the beginnings in university careers present the greatest risk of desertion, because at this stage students confront their expectations with the reality of their academic knowledge, making evident the existence of a gap between the competition of the student in secondary education for recent graduates and the requirements of higher education for freshmen, increasing the risk of desertion and repetition. (Barrios & Caceres, 2011). To mitigate this situation the universities have taken as a measure to offer the new student, courses that in some way support and help to improve the academic weaknesses that may present. (O'Connor & Morrison, 1997) (Rubí, Moreno, Pou, & Jordán, 2010). These courses are commonly known as remedial mathematics or preparation mathematics or pre-basic mathematics, and the University of Cartagena is no stranger to this phenomenon. (Rivas & Saad, 2009) (Sobrepena & Petilos, 2012).

The outlook is not much different from the past. In 1997 O'Connor and Morrison claimed:

In the last twenty years, the number of undergraduate students who took remedial math courses has increased dramatically. For example, between 1975 and 1980, enrollment in remedial math courses at postsecondary institutions increased by 72%, while total student enrollment increased by only 7% (Coleman and Selby, 1982, cited in Chang, 1983). Researchers also suggest that growth in recovery programs can be attributed to the growing diversity of university populations (Cohen, 1984; Tomlinson, 1989).

Some of the research examining the impact of remediation programs at the post-secondary level suggests that remediation courses in math are useful. For example, Kolzow (1986) investigated the school achievement of randomly selected groups of remedial students. The outcomes indicated that grade point average and length of stay in college correlated positively with performance in remedial courses.
Specifically, approximately 44% of students who earned an A in corrective math passed higher-level math courses, while fewer than 4% of those who failed or did not complete the remedial program evidenced the same level of achievement.

Other research found that only 19% of freshmen entering a teacher training program passed a mandatory exam in mathematics (Lee, Lee and Davidson, 1985). Corrective math instruction was provided to improve students' chances of successfully completing the program. The results indicated that remediation significantly improved the mathematical ability of the students.

Finally, the results of a national survey of post-secondary academic institutions suggest that students benefit from remedial education (Chang, 1983). More than 70% of the schools surveyed indicated that the majority of their remedial students successfully completed at least one college-level mathematics course. (O'Connor & Morrison, 1997).

2. Methodology.

2.1 Selection of the population

The dataset encompasses the complete cohort of students enrolled in the Mathematics Bachelor's Program at the University of Cartagena between 1994 and 2016. From this population, academic performance data pertaining to the first and second semesters of the Differential Calculus course were extracted. The selection was conducted in alignment with the four curricular reforms implemented during this period, the specific characteristics of which are outlined below:

- Reform A: The subject of Differential Calculus was located in the first semester and did not have a course prior to this one.

- Reform B: The subject of Differential Calculus was located in the second semester, and in the first semester the course of operative mathematics was offered as a preparatory course.

- Reform C: The subject of Differential Calculus was located in the first semester and did not have a course prior to this one.

- Reform D: The Differential Calculus course was located in the first semester and before starting the course, an extracurricular remedial course was offered, which was sometimes continued in parallel.

At the University of Cartagena, student grades are assigned on a numerical scale ranging from 0.0 to 5.0. For the purpose of evaluating academic performance, the institution employs the following classification scheme: Excellent (4.5–5.0), Very Good (4.0–4.4), Good (3.5–3.9), Fair (3.0–3.4), Deficient (2.0–2.9), and Insufficient (< 2.0).

For each group corresponding to a specific curricular reform, a representative sample consisting of 200 data points was selected. This sampling strategy was employed to ensure comparability across the different reform periods and to maintain statistical robustness in subsequent analyses.

2.2 Statistical analysis.

For the analysis, different statistical tests were carried out on the notes taken from the subject Differential Calculation in the 4 reforms. Initially normality tests were applied in this case Chi-square and Kolmogorov-Smirnov, using software R which showed that the data do not follow a normal distribution.

Non-parametric Mann-Whitney-Wilcoxon tests were applied using the wilcox_test function of the coin package. (Hothorn, Hornik, van de Wiel, & Zeileis, 2008) and Kruskal Wallys using the function dunn.test (Dinno, 2017) of the software R (R Core Team, 2018) which were used to assess differences in achievement levels between students who took a course and those who did not take a course prior to Differential Calculus. All tests were carried out with a significance level of 0.05.

3. Results and Discussion

3.1. Difference in student achievement level

In addition to statistical significance, the observed effect sizes ranging from approximately 0.24 to 0.31 provide practical insight into the magnitude of the differences among reforms. Although considered small to moderate by conventional standards, their consistency suggests that preparatory mathematics courses may foster a measurable and relevant improvement in student performance. This emphasizes the importance of addressing both statistical and educational relevance when assessing curricular interve...

3.1.1. Mann-Whitney- Wilcoxon Test

The Mann-Whitney-Wilcoxon test was initially applied for the 4 data sets. This test contrasts the probability that an observation of the population X overcomes an observation of the population Y is equal to the probability that an observation of the population Y surpasses one of the populations X. In other words, the values of one population do not tend to be greater than those of another, which is why the null and alternative hypothesis was raised in the following way:

$$H0: P(X > Y) = P\ (Y > X)$$
$$Ha: P(X > Y) \neq P(Y > X)$$

This was taken in accordance with our data as they differ not only from their location but also from their dispersion, variance and symmetry. As can be seen in figures 1 to 12.

As in any statistical test, it is not only necessary to indicate the p-value but also the size of the observed effect, since this will allow us to know if, in addition to being significant, the difference is practical, and to analyses the probability of maintaining the null hypothesis when it would have to be rejected; therefore, the size effect obtained in the comparisons was calculated, as follows $TE = Z/(n1\ y\ n2)$ where $n1\ y\ n2$ is the size of each sample in this case as the samples are the same n1=n2=200.

The statistical analysis was carried out using the Mann-Whitney-Wilcoxon non-parametric test, implemented through the R programming environment. The use of R ensured computational accuracy and reproducibility of the analysis, allowing for a rigorous evaluation of the hypotheses under investigation.

3.1.1.1 Comparison Between Reform A vs. Reform B.

The comparative analysis between the two curricular reforms yielded a p-value of 3.361×10^{-8}, which is well below the conventional threshold of 0.05. This outcome indicates a statistically significant difference between the distributions of the two groups, with a confidence level of 95%. Consequently, the null hypothesis of no difference is rejected for $\alpha = 0.05$.

Furthermore, the calculated effect size was 0.2732, derived from a standardized test statistic of $z = -5.464$. Although the magnitude of this effect size may be interpreted as small to moderate according to conventional benchmarks, it nonetheless confirms the presence of a meaningful difference between the groups, thereby reinforcing the decision to reject the null hypothesis and supporting the assertion that the curricular reforms exerted a measurable impact on student performance.

3.1.1.2 Comparison Between Reform A vs. Reform C.

The statistical comparison conducted between the two curricular reforms yielded a p-value of 0.07181, which exceeds the conventional significance level of 0.05. This indicates that, at a 95% confidence level, there is insufficient statistical evidence to reject the null hypothesis. In other words, the differences observed in student performance across the two curricular models are not statistically significant within the accepted margin of error ($\alpha = 0.05$).

While the data may suggest a potential tendency toward variation, the results do not allow us to affirm the existence of a consistent or systematic effect attributable to the curricular changes. Therefore, the null hypothesis asserting that there is no difference in the probabilities associated with student academic performance between the two groups remains valid within the current analytical context. These findings highlight the importance of conducting further studies, possibly involving larger sample sizes or alternative analytical techniques, in order to more accurately capture any subtle effects that may not have been detected in the present evaluation.

3.1.1.3 Comparison Between Reform A vs. Reform D.

The results obtained when comparing these two reforms showed that the p-value is less than 0.05 (p-valor = 1.022e-06), therefore, there is a statistically significant difference between the probabilities with a confidence level of 95.0%. Which implies that the null hypothesis is rejected for α= 0,05.

The result of Z = -4.8489, we take it to calculate the size of the observed effect obtaining the value 0.242445, this indicates us that there is not a significant probability to maintain the null hypothesis.

3.1.1.4 Comparison Between Reform B vs. Reform C.

The results obtained when comparing these two reforms showed that the p-value is less than 0.05 (p-valor = 4.298e-10), therefore, there is a statistically significant difference between the probabilities with a confidence level of 95.0%. Which implies that the null hypothesis is rejected for α = 0,05.

The result of Z = 6.1577, we take it to calculate the size of the observed effect obtaining the value 0.307885, this indicates that there is no significant probability to maintain the null hypothesis.

3.1.1.5 Comparison Between Reform B vs. Reform D.

The results obtained when comparing these two reforms showed that the p-value is greater than 0.05 (p-valor = 0.7871), therefore, there is no statistically significant difference between the probabilities with a confidence level of 95.0%. Which implies that the null hypothesis is not rejected for α = 0, 05.

3.1.1.6 Comparison Between Reform C vs. Reform D.

The results obtained when comparing these two reforms showed that the p-value is less than 0.05 (p-valor = 3.043e-09), therefore, there is a statistically significant difference between the probabilities with a confidence level of 95.0%. Which implies that the null hypothesis is rejected for α= 0,05.

The result of Z = -5.8571, we take it to calculate the size of the observed effect obtaining the value 0.292855, this indicates us that there is not a significant probability to maintain the null hypothesis.

This outcomes shows that there is sufficient evidence to affirm that the values of reform B and D are more likely to be above the values of reform A and C. This result shows that there is sufficient evidence to affirm that the values of reform B and D are more likely to be above the values of reform A and C. This result shows that there is sufficient evidence to affirm that the values of reform B and D are more likely to be above the values of reform A and C. This implies higher grades for students who developed a course prior to Differential Calculus.

On the other hand, this result shows that there is sufficient evidence to affirm that the values of Reform A have the same probability of distancing that the values of Reform C, similarly the values of Reform B have the same probability of distancing that the values of Reform D. This implies that:

a. The students who took the subject Differential Calculus in Reform A or C who did not develop a course prior to this one obtained statistically the same results in spite of being in different reforms.

b. The students who took the subject Differential Calculation in reform B or D who developed a course prior to this one obtained statistically the same results in spite of being in different reforms.

To enhance interpretability, the graphical results should be examined in the logical order of comparison:

- Figures 1 & 2: Reform A vs B
- Figures 3 & 4: Reform A vs C
- Figures 5 & 6: Reform A vs D
- Figures 7 & 8: Reform B vs C
- Figures 9 & 10: Reform B vs D
- Figures 11 & 12: Reform C vs D

This sequential arrangement allows for coherent visualization and understanding of the shifts in grade distributions. Reforms B and D show rightward shifts in boxplots and increased density in higher-score regions, suggesting better performance with preparatory courses. Conversely, the overlapping distributions in comparisons without remediation indicate stagnant outcomes.

It is worth noting that Reform D, despite being extracurricular, yields effects comparable to the formally embedded Reform B, hinting that the intensity and focus of the intervention may be more critical than its formal curricular status. This warrants further research into the design and implementation of effective preparatory modules.

3.1.2 Graphical Results

In Figures 1, 5, 7 and 11 it is observed that Reforms B and D have a displacement to the right (higher notes) for the data group which contains a mathematics course prior to Differential Calculus in comparison with Reforms A and C.

In Figures 2, 6, 8, and 12 it can be seen that for reforms B and D the data have higher density at higher values compared to reforms A and C.

Figures 3 and 9 show that reforms A and B do not have a significant displacement compared to reforms C and D respectively.

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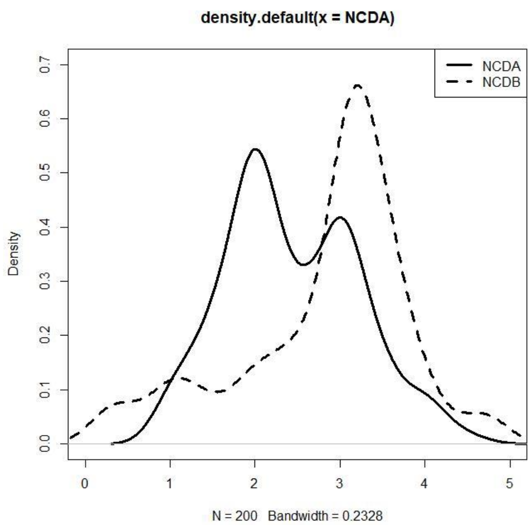
Figures 4 and 10 show that for Reform A and B the data do not statistically have a difference in density at values compared to Reforms C and D respectively.



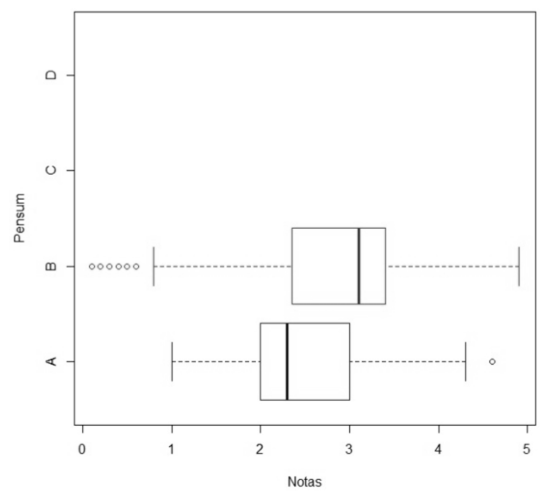
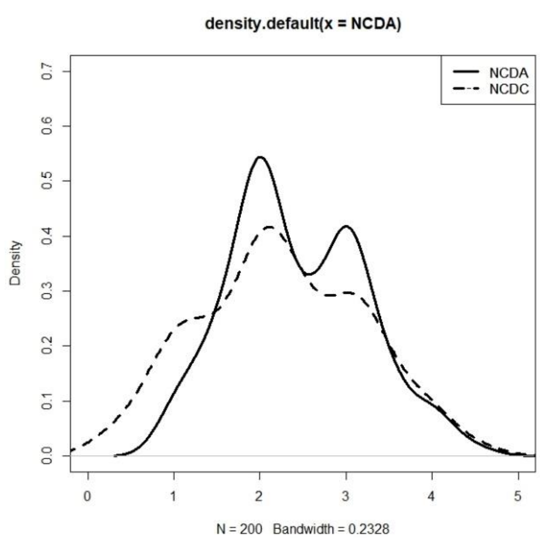


Figure 1. Box and whisker diagram of Reforms A and B. **Figure 2.** Density diagram of reforms A and B



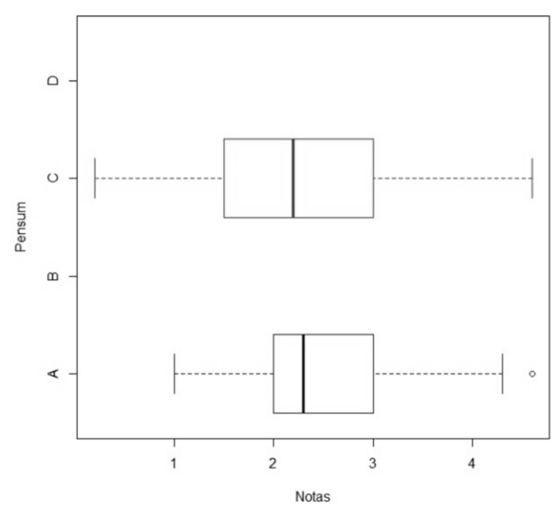
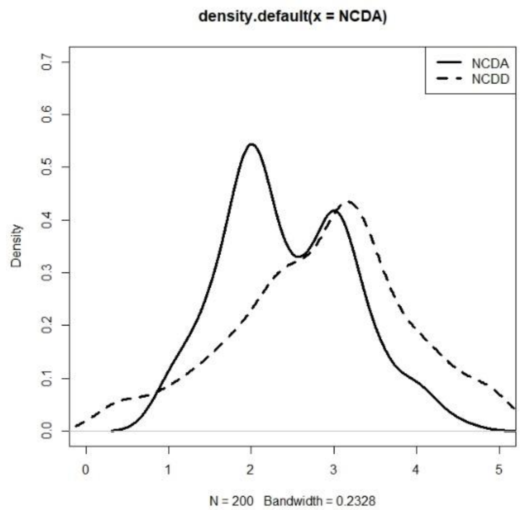


Figure 3. Box and whisker diagram of Reforms A and C. **Figure 4.** Density diagram of Reforms A and C.



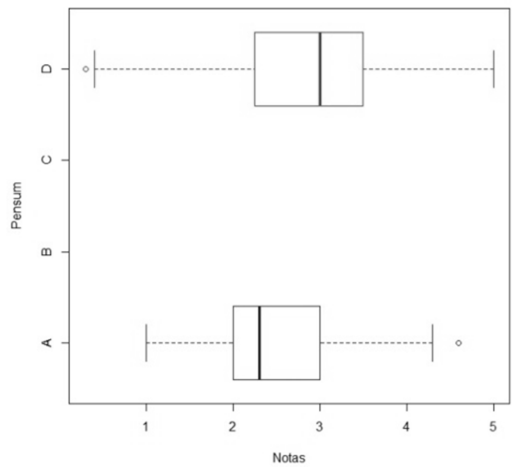
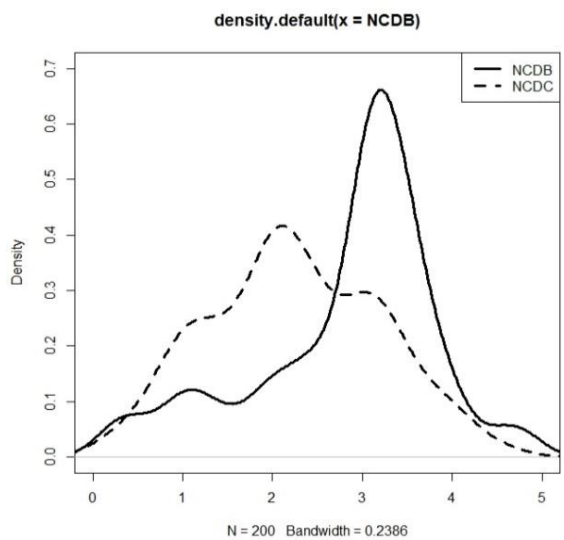


Figure 5. Box and Whisker Diagram of Reforms A and D **Figure 6.** Density diagram of Reforms A and D



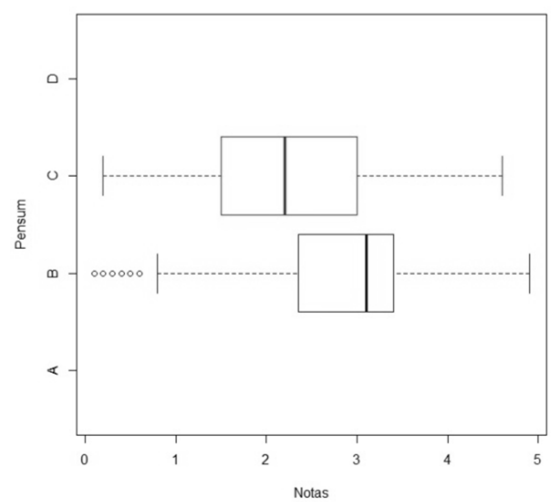


Figure 7. Box and Whisker Diagram of Reforms B and C **Figure 8.** Density diagram of reforms B and C

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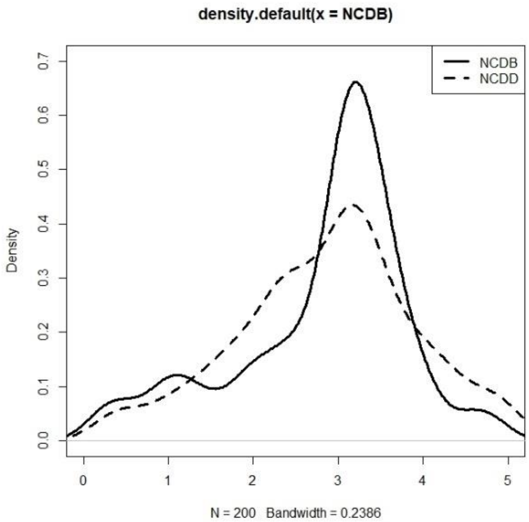


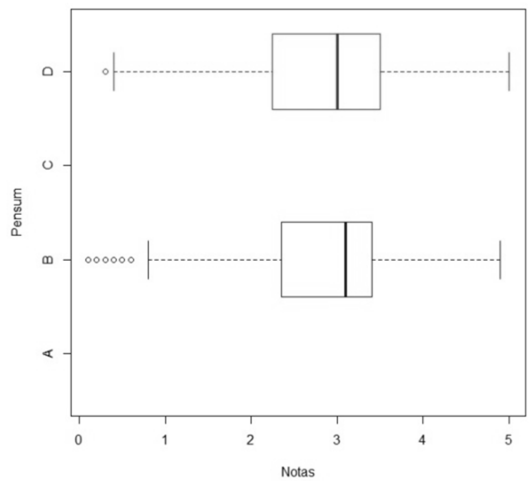
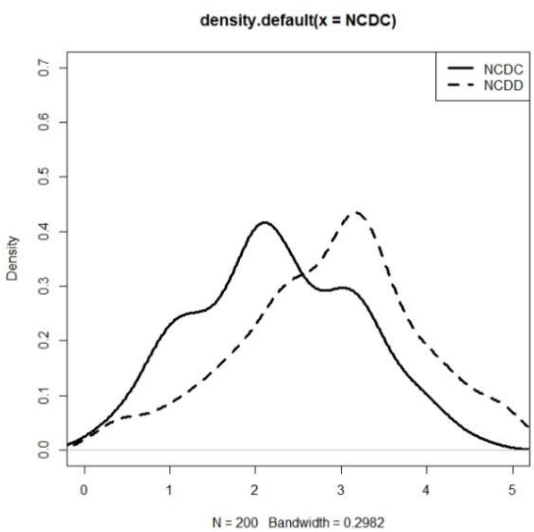


Figure 9. Box and whisker diagram of reforms B and D **Figure 10.** Density diagram of reforms B and D



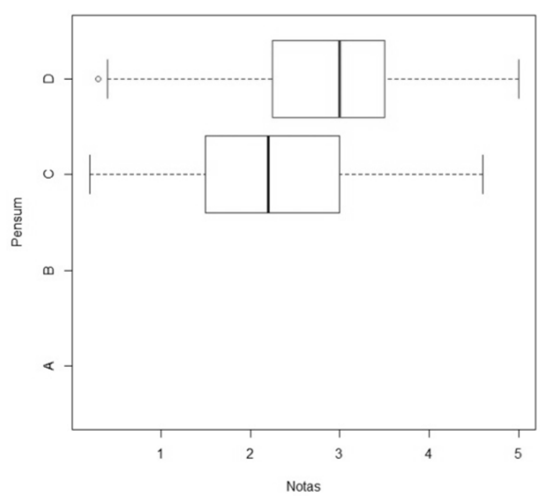


Figure 11. Box and moustache diagram of reforms C and D **Figure 12.** Density diagram of reforms C and D

3.1.3 Test Kruskal - Wallis - Dunn´s

This test was applied in order to compare simultaneously the 4 reforms under study and also to make comparisons two to two, since the method is sensitive to repetitions of data was added Bonferroni's correction.

3.1.3.1 General comparison of Reforms A, B, C and D.

We applied the method of Kruskal - Wallis - Dunn´s with Bonferroni adjustments and obtained $\chi^2 = 64.2694$ with 3 degrees of freedom and $p - value = 0$. The comparisons of the reforms are listed below:
When comparing reform B with reform A, we obtain a value Z=-5.102086 which has the p-value 0 associated with it, this indicates that the null hypothesis is rejected. A similar result is obtained when comparing reform C with reform B since it obtains a value of Z= 6.429768 an associated p-value of 0, reform D with reform A whose value of Z= -4.739400 has the associated p-value of 0 and finally reform D with reform C which has a value of Z= -6.067082 and associated p-value of 0.

On the other hand, when comparing reform C with reform A which have the value of Z= 1.327681 and the associated p-value of 0.5528, this indicates that the null hypothesis is not rejected, similarly, reform D with reform B whose value of Z= 0.362685 has the associated p-value of 1.0000.

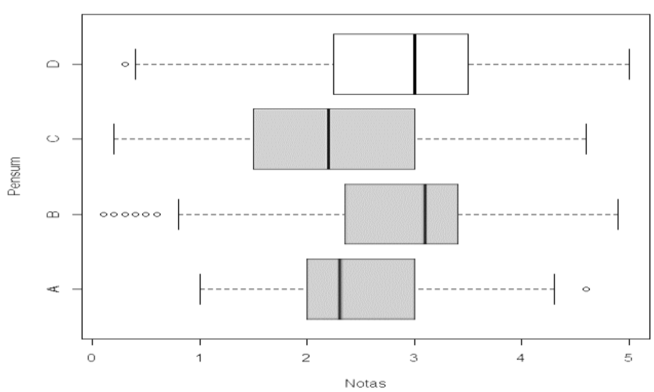


Figura 13. Diagrama de caja y bigote de las 4 reformas A, B, C y D.

The analysis shows that null hypotheses are discarded when comparing Reforms A with B, A with D, C with B and C with D and null hypotheses are accepted in the comparisons of Reforms C with A and D with B. This confirms the results obtained previously when comparisons were made using the Mann-Whiteney-Wilcoxon tests.
While the study focuses solely on final course grades, it opens avenues for broader investigations. Key variables such as students' socioeconomic background, type of prior schooling, admission exam scores, and psychological readiness could enrich the analysis.

Additionally, future studies should compare remedial courses with other pedagogical innovations such as flipped classrooms, peer tutoring, or digital learning environments. Implementing multivariate or longitudinal models could clarify the long-term academic trajectories of students exposed to different types of remediation.

From an institutional perspective, the results support the integration of remedial mathematics as a credited, structured component of the curriculum. Institutions aiming to reduce attrition and enhance early academic performance should prioritize systematic preparatory courses, ideally supported by diagnostic assessments and continuous academic monitoring.

4. Discussion.

The results obtained in this study reveal more than a mere statistical contrast between curricular structures; they shed light on the profound implications of scaffolding learning in mathematically demanding academic programs. The marked improvement in academic performance among students who undertook a preparatory or remedial course prior to Differential Calculus (Reforms B and D) supports the hypothesis that prior exposure to foundational mathematical concepts equips students with cognitive and procedural tools essential for success in higher-level courses.

This aligns with theoretical frameworks in mathematics education, such as Vygotsky's Zone of Proximal Development, which emphasizes the importance of guided support in bridging the gap between what a learner can do independently and what they can achieve with assistance. The preparatory courses in Reforms B and D may serve precisely as that scaffold reducing anxiety, enhancing mathematical confidence, and allowing for a more gradual transition into the abstract reasoning required in calculus.

Moreover, the parity in results between Reforms B (curricular) and D (extracurricular but intensive) invites a nuanced discussion about the formality of instructional interventions. While institutional frameworks often prioritize formally credited courses, these findings suggest that well-structured and purposeful extracurricular programs can yield comparable outcomes. This challenges the traditional dichotomy between formal and informal education, emphasizing instead the role of content quality, instructional design, and engagement.

Another important consideration is the systemic inequality that often underlies academic underperformance in mathematics. Students entering university programs come from diverse socioeconomic and educational backgrounds. Without targeted interventions such as remedial courses, curricular structures risk perpetuating inequities, disadvantaging students who may not have had adequate secondary preparation. Therefore, the implementation of preparatory modules is not merely an academic strategy but also an equity-oriented policy decision.

It is also worth noting that this study exclusively analyzed final grades in Differential Calculus. While valuable, this outcome measure provides a limited perspective on the broader learning process. Academic performance is shaped by a complex interplay of factors including motivation, learning strategies, prior knowledge, and support systems. Future studies should adopt a more holistic approach, integrating qualitative data from student surveys, instructor feedback, and classroom observations to provide a more complete picture of the learning dynamics.

Additionally, further investigation could explore the longitudinal effects of remedial instruction. Do students who begin with a remedial course perform better not only in calculus, but also in subsequent courses such as Integral Calculus, Linear Algebra, or Differential Equations? If so, this would strengthen the argument for early intervention and reinforce the value of curricular coherence throughout the program.

Finally, the discussion must include institutional sustainability. Designing, implementing, and maintaining high-quality remedial programs requires commitment, training, and allocation of resources. This implies that educational policy should integrate these findings into broader strategic planning ensuring that academic support systems are embedded within the fabric of university programs and not treated as ancillary or temporary measures.

5. Conclusions

The findings of this study provide solid empirical evidence that curricular reforms incorporating preparatory mathematics courses prior to the Differential Calculus subject have a positive and statistically significant impact on students' academic performance. This effect, although moderate in size, is consistently observed across the comparative analyses, which reinforces the educational relevance of designing well-structured leveling strategies in higher education programs particularly in disciplines that historically show high levels of academic difficulty and dropout, such as mathematics.

Specifically, students enrolled under Reforms B and D both of which included some form of remedial mathematics achieved better academic outcomes than their peers in Reforms A and C, where no such preparatory structure was implemented. This contrast was not only statistically significant but also educationally meaningful, as it was reflected in a shift from fair/deficient to good/very good performance categories.

Furthermore, the equivalence of performance between B and D, despite the former being a formal curricular course and the latter an extracurricular intensive intervention, suggests that the key factor may not be the nature of the remedial course itself, but rather its existence, intensity, and alignment with the cognitive demands of the Calculus course. This opens an important discussion for academic planning and institutional policy: the necessity of early intervention mechanisms that are flexible, inclusive, and contextually adapted.

Given the structure of the study, which was limited to the final grades in a single subject, future research should expand the scope to include longitudinal tracking of student performance, analysis of dropout rates, and integration of socio-demographic and psychopedagogical variables. Moreover, comparing remedial instruction with other didactic approaches such as active learning environments, peer-led team learning, or blended learning would allow a more comprehensive understanding of the mechanisms that contribute to academic success.

From a curricular and institutional standpoint, the results support the implementation of a formal, credited remedial mathematics course in the early stages of the Mathematics Program. Such a course would not only serve as an academic bridge but also as a mechanism to reduce performance gaps, increase student retention, and promote more equitable learning opportunities.

In sum, the integration of preparatory mathematics as a strategic component of the curriculum is not just a remedial measure but a proactive educational investment that aligns with both student needs and institutional quality goals.

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