LEX LOCALIS-JOURNAL OF LOCAL SELF-GOVERNMENT
ISSN:1581-5374 E-ISSN:1855-363X
VOL. 23, NO. S6(2025)

ANALYSIS FOR THE DYNAMICS OF CONTROL OF CRIME AGAINST WOMEN IN INDIA: AN OPTIMIZATION CONTROL STRATEGY

Shelly Khurana[1,*], Anil Kumar Nishad[2], Priyank Sharma[3]

[1,2]Division of Mathematics, School of Basic Sciences, Galgotias University, Greater Noida, India-201310
[3]Agribusiness and Post harvest supply chain management, AIHSR, Amity University, Noida India-201313

*Corresponding author: Shellykhurana235@gmail.com[1]

ABSTRACT
In this study, a non-linear mathematical model for the dynamics of crime against women in India has been considered based on the principles of optimal control theory. Four time-dependent control mechanisms are incorporated in the present study to curb (minimize) the growing rate of susceptible and hence victim population and enhance (maximize) the crime quitting rate of criminals. These functions reflect the susceptible population's restriction-reduction plan and crime prevention approach to the criminal population. The Pontryagin's maximum principle is used to solve the described optimum control functions. Consequently, the optimal strategies for the control variables have been determined. Numerical findings for several situations are presented to illustrate the effect of controls on the number of victims and criminals. The numerical simulation shows that when the control measures are implemented, there is a noticeable decrease in the number of victims and criminals in comparison to the scenario of no control. Thus, we conclude that to reduce the number of victims and criminals, the government should fund awareness campaigns, enact laws that work, install security cameras, provide emergency devices to susceptible, teach them self-defense techniques, and encourage criminals to change their ways.

Keywords-component; Mathematical model, Crime against Women, Epidemic approach, Stability analysis, Optimal Control.

Introduction

Crime against women is a serious social problem that crosses boundaries, and India is no exception. Despite progress in many areas, the country still faces the threat of violence, discriminate on, and exploitation against women. According to NCRB analysis [1], crimes against women increased by 4% in 2022 over 2021, suggesting a terrifying uptick. The data shows a worrisome increase with a startling 4.45 lakh cases reported in 2022 alone–nearly 51 FBI reports every hour. As per the report, the most common offenses against women under the Indian Penal Code were cruelty committed by husbands or their families (31.4%), followed by kidnapping and abduction (19.2%), assaults on women to offend their modesty (18.7%), and rape (7.1%). The realization of women's and girl's human rights as well as equality, progress, and peace continue to be hindered by such violence against women. To establish a society where women have equal rights and opportunities and may live fearlessly, this issue must be addressed. To guide how to restrict the spread of crime, mathematical models have been essential in deepening our understanding of the fundamental mechanisms that drive it [2-14]. When making decisions involving complicated dynamical systems, optimal control theory is a potent mathematical tool [15]. Several researchers have implemented this theory in their work [16-23]. Optimal control theory has recently been applied to create the best possible intervention plans for a range of "contagious" social illnesses, such as criminality, that are mostly propagated through interaction with afflicted peers. To the best of our knowledge, no research has been done on crime against women in India using the dynamic model and the optimum control technique. Consequently, we will refer to several mathematical models of crime that share similar features and have utilized the optimal control theory. Ibrahim et. al. introduced an optimal control model that accounts for the fluctuations in the subpopulation within the criminal gang population dynamics. In a resource-constrained environment, the model's two control functions describe the susceptible crime deterrence method and the criminal gang population's case identification control. It concluded that the corrective measure including arresting and punishing of criminals is the least expensive and most effective technique in a limited-resource situation [24]. Mushayabasa developed a compartmental model to study the relationship between unemployment and property crime, recognizing that the transition from non-criminal to criminal conduct is socially infectious [25]. He then extended it to the optimal control model by considering three time-dependent control functions representing a potential government initiative or policy meant to lower unemployment and property crime. It was concluded that a nation's property crime and unemployment rates can be efficiently controlled or eliminated with the help of such control measures. Comissiong and Sooknanan reviewed several research papers to analyze optimal control in social models including crime models [26].

We previously presented a mathematical model that examined the dynamics of crime against women in an open population [7]. The following assumptions were made: (i) women become victims while connecting with criminals, (ii) non-susceptible people engage in criminal action while under the influence of criminals; (iii) the flow of criminals to non-susceptible people as a result of behavioral change/laws or police. We also performed parameter estimation to determine the appropriate parameter value and used the least square curve fitting approach, which provides a good match to the data collected from NCRB from 1992 to 2019 [1]. Here, we are expanding our model by incorporating four time-dependent control measures on the target population by restricting their access to hotspot areas, on the target criminal interaction by implementing awareness programs and enforcing laws, on the susceptible criminal interaction by imposing more security measures, and on criminals by encouraging them to quit crime. We investigated the optimal control model using stability analysis. Our goal is to reduce the number of victims and criminals, and we discovered that the combination of the last three control measures produced the best results. Building a future where women's safety and dignity are valued and safeguarded will need ongoing work.

The paper is organized as follows: an optimal control model is formulated in Section 2. Section 3 analyzes the optimal control model by identifying the prerequisites for optimal control's existence. Section 4 provides the necessary conditions for optimality. Section 5 conducts the numerical simulation and discussion. The conclusion is found in Section 6.

I. FORMULATION OF OPTIMAL CONTROL MATHEMATICAL MODEL

For the formulation of the model, we are dividing the total population $N(t)$ into four distinct compartments, namely, targeted population $X(t)$, susceptible women population $Y(t)$ (likely to become a victim), criminals $C(t)$ and women victims $V(t)$. Hence,

$$N(t) = X(t) + Y(t) + C(t) + V(t). \qquad (1)$$

We assume that the women population is likely to become victims and hence join the susceptible class with a rate proportional to the non-susceptible population i.e., $\gamma_1 X$. Women from the susceptible class move to the victim class when they interact with criminals at a rate proportional to the density of susceptible women and the density of criminals i.e., βCY. Furthermore, it is assumed that non-susceptible committing crimes under the influence of criminals and enter the criminal class with a rate proportionate to both the density of non-susceptible and the density of criminals i.e., $\gamma_2 XC$. Also, there is a possibility of criminals giving up crime and moving to non susceptible class with a rate proportional to the criminal population i.e., $\gamma_3 C$. Lastly, the natural death rate μ of all classes is considered to be proportional to their respective class density.

Considering all the abovementioned assumptions, the following mathematical model is proposed:

$$\begin{aligned}\frac{dX}{dt} &= A - \gamma_1 X - \gamma_2 XC + \gamma_3 C - \mu X,\\ \frac{dY}{dt} &= \gamma_1 X - \beta CY - \mu Y,\\ \frac{dC}{dt} &= \gamma_2 XC - \gamma_3 C - \mu C,\\ \frac{dV}{dt} &= \beta CY - (\mu + \alpha)V,\end{aligned} \tag{2}$$

where $X(0) > 0, Y(0) \geq 0, C(0) \geq 0, V(0) \geq 0$.

In the above model system, A is the immigration rate, parameter γ_1 measures the rate of flow from non-susceptible to susceptible, γ_2 is the crime committing rate of non-susceptible, γ_3 is the conversion rate of criminals to non-susceptibles, β is the rate at which susceptible are getting victimized under the influence of criminals, α is the death rate of victims and μ is the natural death rate.

To reduce crime against women, we are introducing four time-dependent control functions $u_1(t),u_2(t),u_3(t),u_4(t)$ in model 2. Here, the control variable $u_1(t)$ is defined to be restriction control, which restricts the target population from going into the risk area and becoming susceptible. This can be done by avoiding crime hotspot areas. The second control variable $u_2(t)$ is the reduction control, which reduces the interaction between the target and criminals via practicing laws or awareness programs etc. The interaction between susceptible and criminals can be reduced through the police force or via technology i.e., imposing more security cameras, providing emergency help devices to susceptible, by teaching them self-defense techniques. To study the impact of such measures, the model incorporated the third control variable $u_3(t)$. The fourth control variable $u_4(t)$ supports a reduction in criminals through awareness programs, behavioral changes in them or scared by law enforcement. The above discussion leads to the following optimal control model:

$$\begin{aligned}\frac{dX}{dt} &= A - \gamma_1(1 - u_1)X - \gamma_2(1 - u_2)XC + \gamma_3(1 + u_4)C - \mu X,\\ \frac{dY}{dt} &= \gamma_1(1 - u_1)X - \beta(1 - u_3)CY - \mu Y,\\ \frac{dC}{dt} &= \gamma_2(1 - u_2)XC - \gamma_3(1 + u_4)C - \mu C,\\ \frac{dV}{dt} &= \beta(1 - u_3)CY - (\mu + \alpha)V,\end{aligned} \tag{3}$$

The problem is to minimize the objective functional:

$$M[u_1,u_2,u_3,u_4] = \int_0^T \left[C(t) + V(t) + \frac{\theta_1}{2}u_1^2 + \frac{\theta_2}{2}u_2^2 + \frac{\theta_3}{2}u_3^2 + \frac{\theta_4}{2}u_4^2\right] dt \tag{4}$$

where T is the final time and, $\theta_1,\theta_2,\theta_3$ and θ_4 are the cost coefficients.

Here, we need to find optimal controls u_1^*,u_2^*,u_3^*,u_4^* such that

$$M\left(u_1^*,u_2^*,u_3^*,u_4^*\right) = \min M\{(u_1,u_2,u_3,u_4):u_1,u_2,u_3,u_4 \in U\}$$

where $U = \{(u_1,u_2,u_3,u_4) \in L^2[0,T]\}$ is the control set such that u_1,u_2,u_3,u_4 are measurable and $0 \leq u_i \leq 1$ for $t \in [0,T]$ for i=1,2,3,4.

II. EXISTENCE OF OPTIMAL CONTROL

The existence of optimal control is demonstrated in this section.

- **Theorem 1**. *A set* $u^* = \left(u_1^*,u_2^*,u_3^*,u_4^*\right) \in U$ *of optimal control variable exists such that*

$$M\left(u_1^*,u_2^*,u_3^*,u_4^*\right) = \min M\{(u_1,u_2,u_3,u_4)|u_1,u_2,u_3,u_4 \in U\}$$

- *Proof.* The following steps need to be fulfilled for the existence of optimal control.
  - There should be some value in the collection of controls and associated state variables. The findings of Fleming and Rishel [27] and Boyce and DiPrima ([28], Theorem 7.1.1) can be used to demonstrate this.
  - The functional M satisfies the required convexity.
  - The set U is both convex and closed.
  - The optimal system (3) is bounded above by the linear function in the state and control variable.
  - The objective functional (4) has convex integrand in control set U, which is bounded below by $\tau_1 + \tau_2\left(\sum_{i=1}^{4}|u_i|^2\right)^{\beta/2}$ where τ_1,τ_2 and $\beta > 1$ are constants, i.e,

$$L(C,V,u_1,u_2,u_3,u_4) \geq \tau_1 + \tau_2\left(|u_1|^2 + |u_2|^2 + |u_3|^2 + |u_4|^2\right)^{\frac{\beta}{2}}$$

This might alternatively be written as

$$C(t) + V(t) + \frac{\theta_1}{2}u_1^2 + \frac{\theta_2}{2}u_2^2 + \frac{\theta_3}{2}u_3^2 + \frac{\theta_4}{2}u_4^2$$
$$+ \geq \tau_1 + \tau_2\left(|u_1|^2 + |u_2|^2 + |u_3|^2 + |u_4|^2\right)^{\frac{\beta}{2}}$$

The condition is verified by the fact that state variable are bounded and by taking $\tau_1 = 2\inf_{t\in[0,T]}(C(t) + V(t)), \tau_2 = \inf\left(\frac{\theta_1}{2}, \frac{\theta_2}{2}, \frac{\theta_3}{2}, \frac{\theta_4}{2}\right)$ and $\beta = 2$. □

Thus, Fleming and Rishel argue that optimal control exists.

III. NECESSARY CONDITION FOR OPTIMAL CONTROL

The necessary optimality conditions can be derived using the Pontryagin's maximum principle [29], which aims at converting the optimal system (3) and objective functional (4) into the problem of minimizing Hamiltonian function H, given by:

$$H = C(t) + V(t) + \sum_1^4 \frac{\theta_i}{2}u_i^2 + \sum_1^4 \lambda_i(t)S_i \quad (5)$$

where $\lambda_i(t), i = 1,2,3,4$ are the adjoint variables and $S_i, i = 1,2,3,4$ represent the variable on the right side of the differential equation for the i-th state.

Theorem 2. *For the optimal controls* $\left(u_1^*,u_2^*,u_3^*,u_4^*\right)$ *and their corresponding state solutions* X^*,Y^*,C^*,V^**, there exist adjoint functions* $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ *such that*

$$\lambda_1' = \gamma_1(1 - u_1)(\lambda_1 - \lambda_2) + \lambda_1\mu + (\lambda_2 - \lambda_3)\gamma_2(1 - u_2)C$$
$$\lambda_2' = \beta(1 - u_3)C(\lambda_4 - \lambda_2) + \lambda_2\mu$$
$$\lambda_3' = -1 + \gamma_2(1 - u_2)X(\lambda_1 - \lambda_3) - \gamma_3(1 + u_4)(\lambda_1 - \lambda_2) + \beta(1 - u_3)Y(\lambda_2 - \lambda_4)$$
$$\lambda_4' = -1 + \lambda_4(\mu + \alpha)$$

with transversality condition $\lambda_i(T) = 0$ *for* $i = 1,2,3,4$ *and the control variables* $\left(u_1^*,u_2^*,u_3^*,u_4^*\right)$ *satisfy the following optimality conditions:*

$$u_1^* = \min(1, \max(\frac{(\lambda_2 - \lambda_1)\gamma_1 X(t)}{\theta_1})),$$
$$u_2^* = \min(1, \max(\frac{(\lambda_3 - \lambda_1)\gamma_2 X(t)C(t)}{\theta_2})),$$
$$u_3^* = \min(1, \max(\frac{(\lambda_4 - \lambda_2)\beta C(t)Y(t)}{\theta_3})),$$
$$u_4^* = \min(1, \max(\frac{(\lambda_3 - \lambda_1)\gamma_3 C(t)}{\theta_4})).$$

Proof. The Hamiltonian function H is given by:

$$H = C(t) + V(t) + \frac{\theta_1}{2}u_1^2 + \frac{\theta_2}{2}u_2^2 + \frac{\theta_3}{2}u_3^2 + \frac{\theta_4}{2}u_4^2 + \lambda_1[A - \gamma_1(1 - u_1)X - \gamma_2(1 - u_2)XC + \gamma_3(1 + u_4)C - \mu X] + \lambda_2[\gamma_1(1 - u_1)X - \beta(1 - u_3)CY - \mu Y] + \lambda_3[\gamma_2(1 - u_2)XC - \gamma_3(1 + u_4)C - \mu C] + \lambda_4[\beta(1 - u_3)CY - (\mu + \alpha)V].$$

where Pontryagin's maximum principle may be can be employed to compute the adjoint functions, as follows:

$$\frac{d\lambda_1}{dt} = -\frac{\delta H}{\delta X}$$
$$\frac{d\lambda_2}{dt} = -\frac{\delta H}{\delta Y}$$
$$\frac{d\lambda_3}{dt} = -\frac{\delta H}{\delta C}$$
$$\frac{d\lambda_4}{dt} = -\frac{\delta H}{\delta V}$$

Thus, the system of adjoint equations at optimal controls $u_1^*, u_2^*, u_3^*, u_4^*$ and their corresponding state variables X^*, Y^*, C^*, V^* is a follows:

$$\lambda_1' = \gamma_1(1 - u_1)(\lambda_1 - \lambda_2) + \lambda_1\mu + (\lambda_2 - \lambda_3)\gamma_2(1 - u_2)C^*$$
$$\lambda_2' = \beta(1 - u_3)C^*(\lambda_4 - \lambda_2) + \lambda_2\mu$$
$$\lambda_3' = -1 + \gamma_2(1 - u_2)X^*(\lambda_1 - \lambda_3) - \gamma_3(1 + u_4)(\lambda_1 - \lambda_2) + \beta(1 - u_3)Y^*(\lambda_2 - \lambda_4)$$
$$\lambda_4' = -1 + \lambda_4(\mu + \alpha)$$

Further, the optimality conditions for control variables $u_1^*, u_2^*, u_3^*, u_4^*$ can be obtained by the following condition:

$$\frac{\delta H}{\delta u_i} = 0 \text{ for } i = 1,2,3,4$$

that is

$$0 = \frac{\delta H}{\delta u_1} = \theta_1 u_1^* + (\lambda_1 - \lambda_2)\gamma_1 X^* \quad (6)$$
$$0 = \frac{\delta H}{\delta u_2} = \theta_2 u_2^* + (\lambda_1 - \lambda_3)\gamma_2 X^* C^* \quad (7)$$
$$0 = \frac{\delta H}{\delta u_3} = \theta_3 u_3^* + (\lambda_2 - \lambda_4)\beta C^* Y^* \quad (8)$$
$$0 = \frac{\delta H}{\delta u_4} = \theta_4 u_4^* + (\lambda_1 - \lambda_3)\gamma_3 C^*. \quad (9)$$

The optimal control is limited to $u_i^* = \min\{\max(0,u_i),1\}$ to fulfill the required restrictions for the control functions (i.e.,$0 \leq u_i \leq 1$ and $t \in [0,T]$). Thus, we obtain the desired values of optimal controls by applying the equations (6), (7), (8), and (9) along with the boundaries of the controls.

IV. Optimal Control Model Numerical Simulation: Results and Discussion

This section explores how control strategies affect crime against women through numerical simulation. The optimality system may not always be solvable analytically; thus approximations of the solutions and their visualization are achieved by numerical techniques. The optimality system may be thought of two-point boundary system due to the initial state system condition and the terminal state of the adjoint system. Such type of two-point boundary value problem can be solved using the more precise and thorough iterative method namely the Runge-Kutta fourth-order algorithm. A multiple-step approach, the Runge-Kutta method takes a defined set of prior values $x_{n-k},\ldots,x_k$ where n is the number of steps and the solution at time x_{k+1} is found. This technique called the forward-backward sweep approach, is explained in a book by Lenhart and Workman [15]. Starting with a first estimate for the control variable, we use the Runge-Kutta forward sweep technique to approximate solutions for state equations given initial conditions for states. We estimate solutions for adjoint equations given the final time conditions for adjoints and the state solutions from the prior step using the Runge-Kutta backward sweep approach. The value of the control variables is updated by averaging the previous value and the new value obtained from the control characterization. The procedure is repeated for updating the controls and the forward numerical scheme until the convergence of all state, adjoint, and control values is achieved.

By taking time in weeks, we have used MATLAB to write and compile the code where the estimated initial values of state variable are $X = 1000000, Y = 10000, C = 340058, V = 7903711$, the terminal condition for the adjoint system is $\lambda_i(T) = 0$ for $i = 1,2,3,4$, T=50 weeks and the parameter values are given by the following table:

Table 1: Parameters values for numerical simulation of optimal control model.

Parameters	Description	Baseline value (per year)	Source
A	Immigration Rate	13011000	Estimated
γ_1	Rate of flow from X to Y	0.2	Fitted
γ_2	Crime committing rate of X	0.00000000132	Fitted
γ_3	Rate of flow from C to X	0.0115	Fitted
β	Rate at which Y are getting victimized under the influence of criminals	0.000000000240112	Fitted
α	Death rate of victims	0.00005	Fitted
μ	Natural death rate	0.00028	Estimated

The impact of the control measures on crime against women in various combinations is then plotted on the graphs. The scenario in which no optimal control method is used is displayed in all the upcoming figures, to determine the best alternative. Subsequently, we suggest multiple control techniques utilizing diverse combinations of control measures and assessing their efficacy, assisting in the identification of the most appropriate control approach to battle offenders and victims.

a) Control strategy implementing one control at a time.

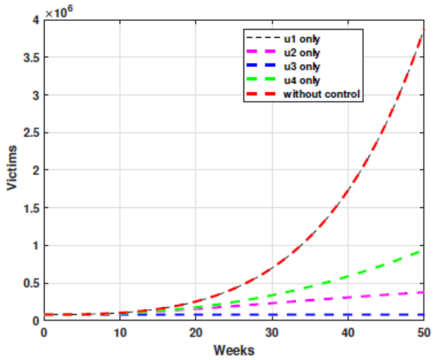

Figure 1: Variation in victims $V(t)$ for different controls combinations.

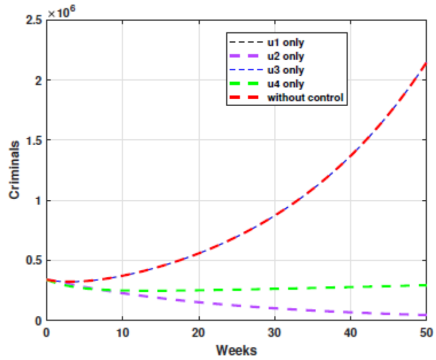

Figure 2: Variation in criminals $C(t)$ for different controls combinations.

Figure 1 displays the variation of victims when control variables u_i's;$i = 1,2,3,4$ are implemented alone. The figure clearly shows that control variable u_1 alone does not affect reducing the victims, however, control variable u_3 most efficiently lowers the victims from 3877690 to 85228 in 50 weeks, making it the ideal control approach to use. Thus, it may be proposed that victims can be effectively minimized by reducing the interaction between susceptible and criminals via installing more security cameras, offering emergency aid gadgets to susceptible, training them in self-defense tactics, and so on. Similarly, figure 2 depicts the variation of criminals for different control variables. Implementing u_1 and u_3 alone does not reduce the criminals whereas the most effective control variable which reduces criminals from 2147110 to 45482 in 50 weeks is u_2. As a result, awareness programs and practicing laws to minimize interaction between targets and criminals are critical in lowering crime.

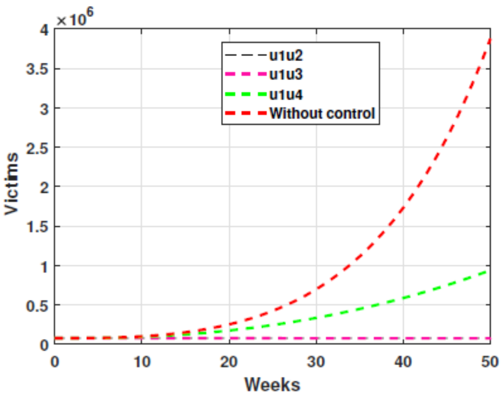

Figure 3: Variation in victims $V(t)$ for different controls combinations.

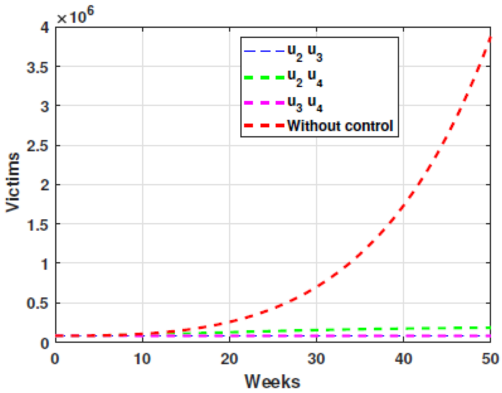

Figure 4: Variation in victims $V(t)$ for different controls combinations.

b) Control strategy implementing two controls at a time:

Figures 3 and 4 represent the variation in victims for several combinations of two control variables at a time. It can be concluded from the two figures that the combination of controls u_2,u_3 and u_3,u_4 have almost identical effects in bringing down the number of victims from 3877690 to 78769 in 50 weeks. Consequently, put any of the following strategies into practice: limiting association between susceptible and criminals by the use of technology or the police along with conducting awareness campaigns to deter target criminal interaction, or encouraging criminals to change their behavior. The variation in criminals considering two control measures at a time is illustrated by figures 5 and 6. It demonstrate that combination of u_2 and u_4 reduces the criminals from 2147110 to 6216 in 50 weeks making it the optimal combination of control measures to implement. In other words, we can say that actions like practicing laws, running awareness campaigns, and behavioral changes in criminals have a significant influence on the decline in the number of criminals.

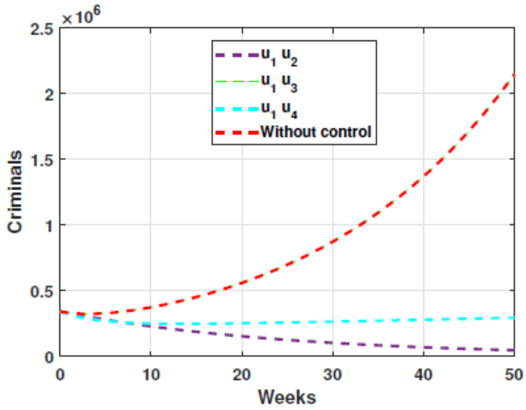

Figure 5: Variation in criminals $C(t)$ for different controls combinations.

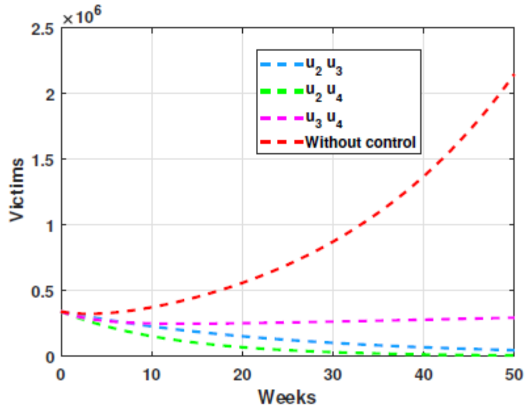

Figure 6: Variation in criminals $C(t)$ for different controls combinations.

c) Control strategy implementing three controls at a time:
Three control measures can be combined in the following four manners: $u_1, u_2, u_3; u_1, u_2, u_4; u_1, u_3, u_4$ and u_2, u_3, u_4. Any of these four strategies can yield almost identical results, however, strategy 4 is found to be more suitable in bringing victims down to 77765 from 3877690 and criminals down to 6173 from 2147110 in 50 weeks. A conclusion can be drawn that practicing laws to prevent target-criminal interaction, enforcing technology to limit susceptible criminal interaction, and motivating criminals to quit crime are shown to be an effective control strategy. The outcome is consistent with applying all four controls together, as will be discussed in the following part.
d) Control strategy implementing all four controls:
The variation in victims and criminals respectively on implementing all four controls together is shown in figures 7 and 8 respectively. It is evident from the figures that encouraging the target population to avoid crime hotspot areas, lowering target-criminal and susceptible-criminal interaction through awareness campaigns and police force/technology respectively, and non-criminalization of criminals through behavioral changes cause victims to decrease from 3877690 to 77765 in 50 weeks while criminals decrease from 2147110 to 6173 in the same time frame.

Although implementing three control variables $u_2, u_3,$ and u_4 and implementing all the control variables have the same impact in reducing victims and criminals, it can be concluded that, of all the cases discussed above, the most effective control strategy for reducing victims and criminals in long run is to implement the three control measures $u_2, u_3,$ and u_4 together, i.e., practice laws to prevent target-criminal interaction, enforce technology to limit susceptible criminal interaction and encourage criminals to stop committing crimes.

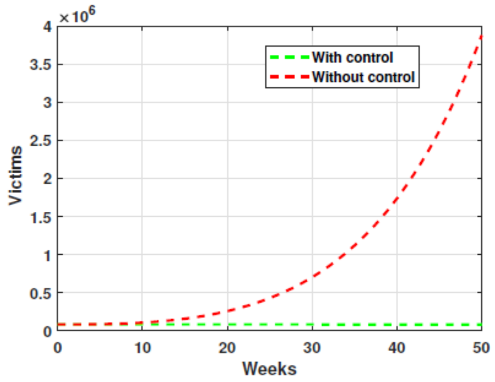

Figure 7: Variation in victims $V(t)$ when all controls measures are implemented.

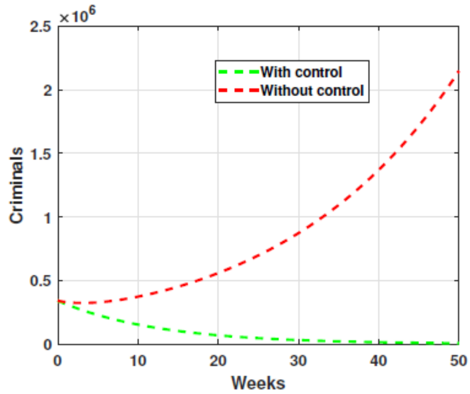

Figure 8: Variation in criminals $C(t)$ when all controls measures are implemented.

V. Conclusion

This work represents the application of optimal control techniques on a dynamical model system with control strategies aimed at minimizing the number of criminals and victims. The adopted control measures include prohibiting the target population from entering the risk area, practicing laws or awareness programs that lessen contact between the target and criminals, minimizing interactions between susceptible individuals and criminals through the use of technology or police force and law enforcement-induced behavioral changes or intimidation which deter criminal activities. By converting a constrained optimization problem to an unconstrained Hamiltonian function and deriving adjoint equations from it, Pontryagin's maximum principle enables us to prove the existence of an optimal control problem and to establish the required conditions for optimality. Lastly, to examine and contrast the effects of the different control strategies, we performed numerical simulations of the resulting control problem. The numerical findings revealed that control techniques including all four control measures and control measures (u_2,u_3,u_4) had a substantial influence on reducing criminals and victims making u_1 less effective. Thus, putting control measures in place like awareness campaigns, passing legislation, installing security cameras, giving susceptible people emergency devices, teaching them self-defense skills, encouraging criminals to change their behavior, and frightening them with the presence of law enforcement could eventually result in fewer victims and criminals.

Acknowledgment

We would like to extend our heartfelt thanks to all of the individuals and the organization that have contributed to the publication of this research paper.

References

[1] Report. National crime records bureau. https://ncrb.gov.in/
[2] Abbas S, Tripathi J P, Neha AA. Dynamical analysis of a model of social behavior: Criminal vs non-criminal population. Chaos Solitons Fractals. 2017; 98:121-129.
[3] Akanni J O, Akinpelu F O, Olaniyi S, Oladipo A T, Ogunsola A W. Modelling financial crime population dynamics: optimal control and cost-effectiveness analysis. International Journal of Dynamics and Control. 2019; 8:531-544.
[4] Gonzlez-Parra G, Chen-Charpentier B, Kojouharov H V. Mathematical modelling of crime as a social epidemic. Journal of Interdisciplinary Mathematics. 2018; 21(3):623-643.

[5] H. Ugwuishiwu C, Sarki D S, Mbah G C E. Nonlinear analysis of the dynamics of criminality and victimisation: A mathematical model with case generation and forwarding. Journal of Applied Mathematics. 2019; 11:1-17.
[6] Khurana S, Shekhar S. Mathematical models studying crime dynamics: A review on adopted approaches. Indian J Forensic Med Pathol. 2022; 14(3 Special):223-230.
[7] Khurana S, Shekhar S. Modelling and analysis of the spread of crime against women in open population. "A Handbook on Emerging Trends in Mathematical Sciences & Computing", Nova Science Publishers, USA. 2024.
[8] Mohammad F, Roslan U A M. Analysis on the crime model using dynamical approach. AIP Conference Proceedings. 2017; 1870(1):040067.
[9] Rivera-Castro M, Padmanabhan P, Caiseda C, Seshaiyer P, BoriaGuanill C. Mathematical modelling, analysis and simulation of the spread of gangs in interacting youth and adult populations. Letters in Biomathematics. 2019; 6(2):1-19.
[10] Shukla J B, Goyal A, Agrawal K, Kushwa H. Role of technology in combating social crimes: A modeling study. European Journal of Applied Mathematics. 2013; 24(4):501-514.
[11] Srivastav A K, Ghosh M, Chandra P. Modeling dynamics of the spread of crime in a society. Stochastic Analysis and Applications. 2019; 37(6):991-1011.
[12] Zengli W, Xuejun L. Analysis of burglary hot spots and near-repeat victimization in a large chinese city. ISPRS International Journal of Geo-Information. 2017; 6(5):148-161.
[13] Zhao H, Feng Z, Chavez C C. The dynamics of poverty and crime. Journal of Shanghai Normal University. 2015; 43(5):486-495.18
[14] Azmil R, Mohd Ariff MF, Razali A F, Azmy S N, Darwin N, and Idris KM. Transforming Physical Crime Scene into Geospatial-based Point Cloud Data, Eng. Technol. Appl. Sci. Res. 2024; 14 (3): 13974–13981.
[15] Lenhart S M, Workman J T. Optimal control applied to biological models. CRC Press. 2007.
[16] Aldila D, Rarasati N, Nuraini N, Soewono E. Optimal control problem of treatment for obesity in a closed population. Int J Math Math Sci. 2014; Article ID 273037:7 pages.
[17] Athithan S, Ghosh M, Li X Z. Mathematical modelling and optimal control of corruption dynamics. Asian Eur J Math. 2018;11(6):1850090.
[18] Bolzoni L, Bonacini E, Soresina C, Groppi M. Time-optimal control strategies in sir epidemic models. Math Biosci. 2017; 292:86.
[19] Chen L, Hattaf K, Sun J. Optimal control of a delayed slbs computer virus model. Physica A. 2015; 427:244-250.
[20] Huo L, Lin T, Fan C, Liu C, Zhao J. Optimal control of a rumor propagation model with latent period in emergency event. Adv Differ Equ. 2015; 54.
[21] Laaroussi A E A, Rachik M, Elhia M. An optimal control problem for a spatiotemporal sir model. Int J Dyn Control. 2018; 6(1):384.
[22] Oh C, Masud M A. Optimal intervention strategies for the spread of obesity. J Appl Math. 2015; Article ID 217808:9 pages.
[23] Iqbal J. Modern Control Laws for an Articulated Robotic Arm: Modeling and Simulation. Eng. Technol. Appl. Sci. Res. 2019, 9(2):4057-4061.
[24] Ibrahim O, Okuonghae D, Ikhile M. Optimal control model for criminal gang population in a limited-resource setting. Int. J. Dynam. Control. 2023; 11:835-850.
[25] Mushayabasa, S. Modeling optimal intervention strategies for property crime. Int J Dyn Control. 2017; 5(3):832-841.
[26] Comissiong DMG, Sooknanan J. A review of the use of optimal control in social models. Int J Dyn Control. 2018; 6(4):1841-1846.

[27] Fleming WH, Rishel RW. Deterministic and stochastic optimal control. Springer, New York. 1975.
[28] Boyce WE, DiPrima RC. Elementary differential equations and boundary value problems. New York: John Wiley & Sons. 2009.
[29] Schattler H, Ledzewicz U. The pontryagin maximum principle: From necessary conditions to the construction of an optimal solution. Interdisciplinary Applied Mathematics, Springer New York. 2012; 38:83-194.