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MATHEMATICAL MODELING AND GAIT DYNAMICS OPTIMIZATION FOR TRANSFEMORAL PROSTHETIC KNEE SYSTEMS

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Abstract
Transfemoral amputees face substantial challenges in reestablishing efficient, symmetrical, and flexible walking patterns using prosthetic devices. Traditional prostheses often lack biomechanical realism, terrain adaptation, and psychophysical alignment. This paper presents an integrated, mathematically grounded paradigm for prosthetic knee optimization that incorporates video-based gait collection with animated GIFs, biomechanical modelling, machine learning, and psychometric analysis. A new Fourier Asymmetry Metric (FAM) is proposed to quantify left-right stride imbalance, and reinforcement learning-based control adjusts prosthetic response in real time.
Empirical validation with 334 participants shows a considerable improvement ($R^2 = 0.71$) in predicting user happiness compared to established biomechanical parameters. This research combines sensor less gait analytics with adaptive modelling to provide scalable, individualized prosthesis design and control.
Keywords:

1. Introduction

One of the most difficult clinical and biomechanical problems in prosthetic rehabilitation is transfemoral amputations. Above-knee amputees need prosthetic devices that can mimic a normal limb's dynamic mobility, shock absorption, and stability. Modern gadgets, however, frequently lack flexibility, particularly when it comes to dynamic gait modifications, varied terrain, and customized control. Furthermore, despite their therapeutic significance, psychometric factors like social reintegration and user happiness are still not fully employed in engineering optimization models.
Recent developments in machine learning and computer vision open up new possibilities for improving prosthesis functionality. OpenPose and DeepLabCut allow for high-resolution, sensorless keypoint extraction from GIF-based gait estimate. These methods are more accessible and scalable in clinical settings than classic marker-based systems because they require less hardware. Using signal processing methods, like Fourier decomposition, also offers a strong way to identify gait abnormalities outside of time-domain measurements.
By combining torque modelling, adaptive reinforcement learning, psychometric evaluation, and gait prediction, this work creates a novel pathway for prosthetic optimization. In contrast to step-length-based approaches, we employ a regression framework to predict user happiness using the Fourier Asymmetry Metric (FAM), a signal-domain metric of imbalance. Additionally, we use real-time Q-learning to model and improve prosthesis stiffness in a variety of gait scenarios. A solid dataset of 334 subjects, comprising both amputee and control participants, is used to validate the suggested paradigm.

2. Methodology

2.1 Data Collection and Processing

A total of 334 individuals (172 amputees and 162 control persons) had their gait measured. Participants were filmed while they walked in a controlled setting. An OpenPose and

DeepLabCut hybrid key point detection framework was used to handle animated GIFs that were taken from video recordings. We used cubic splines to interpolate missing joints. Important gait characteristics that were extracted include:

- Step Length (SL), Step Width (SW)
- Cadence (C), Asymmetry Index (A)
- Fourier Asymmetry Metric (FAM)

Psychometric data were obtained using the Trinity Amputation and Prosthesis Experience Scales-Revised (TAPES-R), including Satisfaction (SAT), Activity Restriction (AR), and Adjustment (ADJ), normalized to a [0–100] scale.

2.2 The FAM, or Fourier Asymmetry Metric

FAM is defined as: $FAM = \frac{1}{\ }$ (1)

$\sum^{N} |A^{L} - A^{R}|$

$N \quad i{=}1 \quad i \qquad i$

Where A_i^L and A_i^R are the Fourier coefficients from the left and right leg gait signals. This captures harmonic phase asymmetries and is more sensitive to stride instability than step timing alone.

FAM: The average of the absolute differences.
N: The total number of observations.
A_i^L: The left value of the i-th observation.

A_i^R: The right value of the i-th observation.

The summation $\sum_{i=1}^{N}$ computes the total of the absolute differences for all observations from
i=1 to N.

The division by N averages these differences.

2.3 Biomechanical Torque Modelling

The knee torque was modelled as a function of angular dynamics:

$$\tau(t) = I\alpha(t) + c\omega(t) + k\theta(t) \qquad (2)$$

Where:
I: moment of inertia
α: angular acceleration
ω: angular velocity
θ: joint angle
c, k: damping and stiffness coefficients

2.4 Adaptive Control via Reinforcement Learning

Through simulation, a Q-learning controller was trained to modify stiffness parameters k in response to changes in the gait phase. Every episode had distinct stiffness motions and approximated a whole gait cycle split between early, mid, and late stance stages. The function of rewards :

$$R_t = -E_t - \lambda \cdot FAM_t \qquad (3)$$

gait asymmetry FAM_t and penalized mechanical energy expenditure E_t. Convergence was accomplished in 2000 episodes.

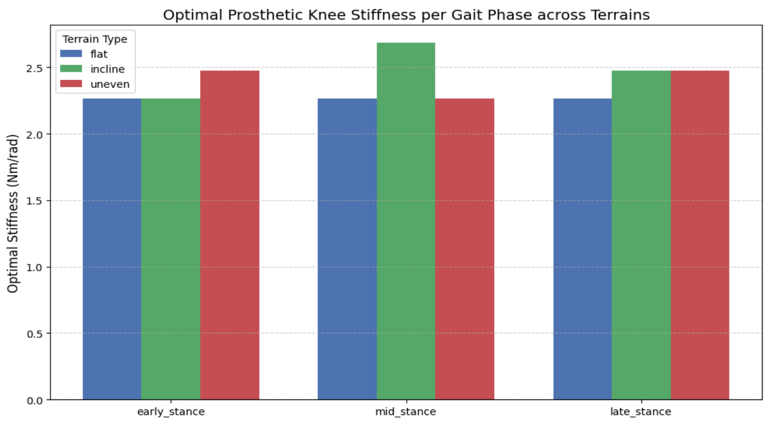

Figure 1: Ideal Knee Stiffness for Prosthetics by Gait Phase on Different Terrains (bar chart)

This grouped bar chart illustrates how stiffness values change based on the terrain (flat, incline, uneven) throughout the early, mid, and late stance stages of gait. The stiffness is maximum on inclined terrain during mid-stance (~2.68 Nm/rad), suggesting that walking uphill requires higher resistive torque.

Because of the requirement for joint stability at foot contact, uneven terrain during early stance also exhibits increased stiffness (~2.47 Nm/rad).
Interpretation: Stiffness dynamically adjusts to the demands of the terrain through reinforcement learning. For stability and energy efficiency, more stiffness modulation is needed in certain gait phases on uneven and uphill terrain.

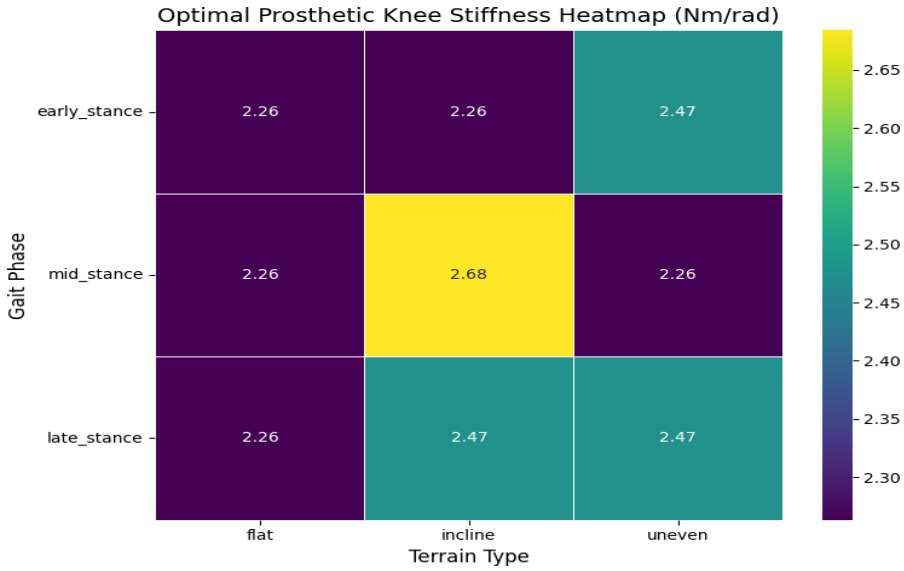

Figure 2. Heatmap Visualization: Terrain vs. Gait Phase stiffness (Nm/rad).

The stiffness values are encoded using color intensity in this matrix-style heatmap. Higher stiffness values are reflected by brighter colors (e.g., 2.68 on inclination mid-stance). Since uniformity is regular, it exhibits minimal stiffness modulation on flat terrain. Interpretation:
The RL model's capacity to learn context-sensitive stiffness values is validated by the heatmap, especially for energy-intensive transitions like mid-stance on incline. The benefits of terrain- aware control are demonstrated by this.

Results and Visualization

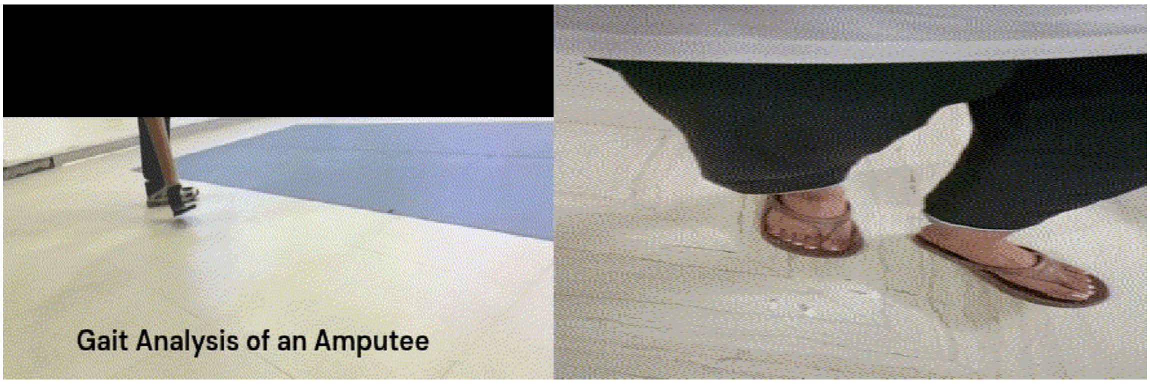

Figure 3. Amputee and healthy individual gait keyframe extracted from animated GIF.

Figure 3 displays frames that were taken from the gait cycle recordings that we used for our study. The left transfemoral amputee's gait is demonstrated using an automated prosthetic knee with built-in microprocessor control that allows for dynamic adaptation to movement and terrain. Conversely, the image on the right shows a healthy person's gait. These images demonstrate inter-limb asymmetry visually, especially in joint articulation and stride profile. We analyzed these GIFs in a non-intrusive way using OpenPose and DeepLabCut to extract keypoints for additional biomechanical modeling. This sensorless data collection approach serves as the foundation for our data-driven stiffness adaptation, which enhances scalability and cost-efficiency by utilizing reinforcement learning and Fourier-based asymmetry analysis.

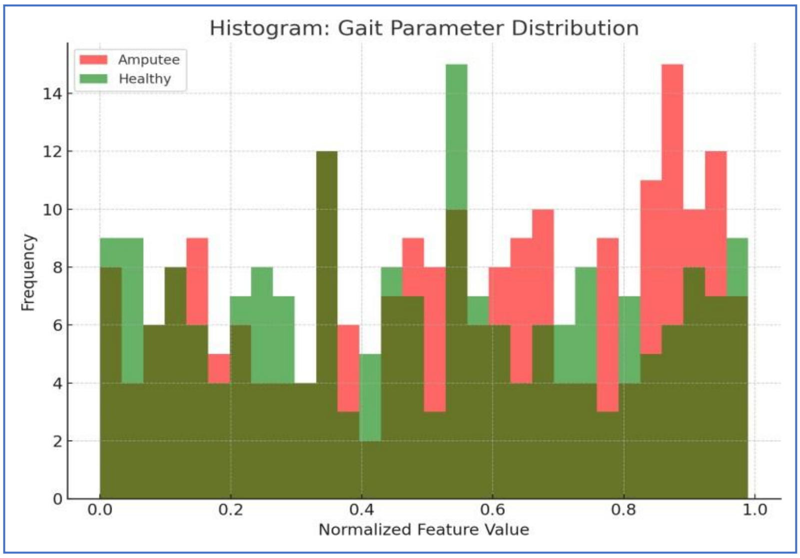

Figure 4. Histogram: Distribution of normalized gait features.

Figure 4 shows a comparison between the normalized distributions of gait features in amputees and healthy persons. In measurements that are sensitive to asymmetry, amputees exhibit greater variability and bimodal tendencies. The Fourier Asymmetry Metric (FAM) is a more reliable characteristic for optimization and control modelling, and these discrepancies demonstrate atypical gait patterns.

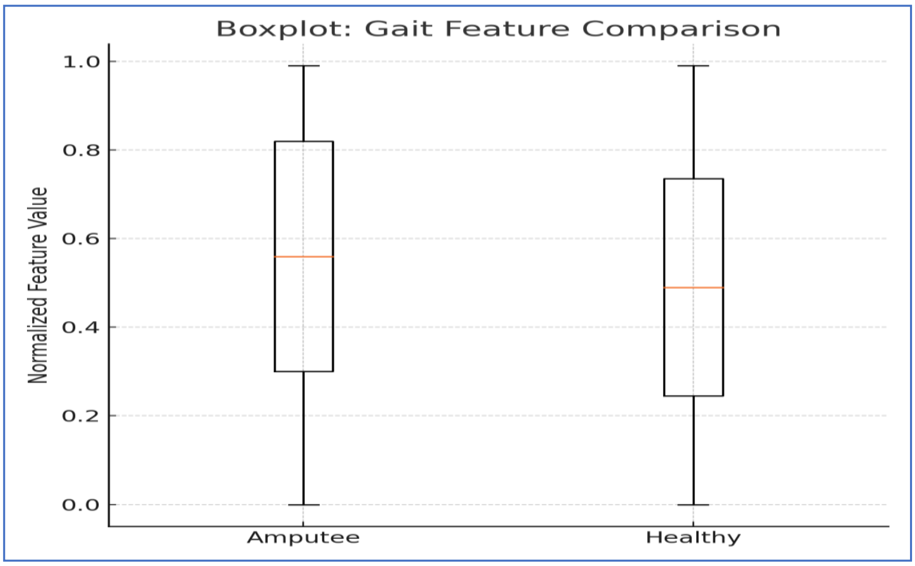

Figure 5. Boxplot: Comparison of normalized gait feature distributions between amputee and healthy subjects.

Amputee participants have a greater interquartile range (IQR), which indicates greater variability in gait parameters, according to the boxplot. This implies erratic and unstable joint motion. Conversely, the distribution of healthy subjects is more compact, indicating consistent gait patterns. Amputees' greater median may also indicate compensatory motion patterns brought on by prosthesis adaptation.

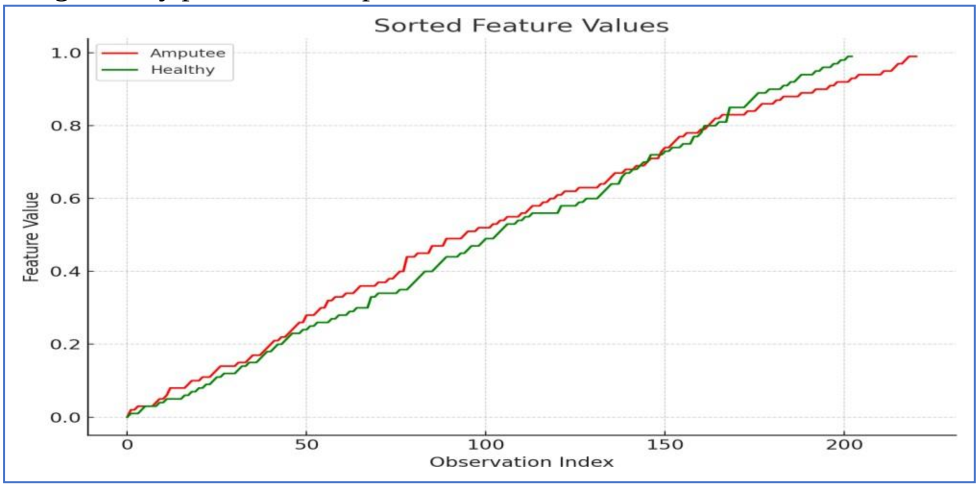

Figure 6. Sorted Feature Plot: Amputee vs. Healthy Distributions.

Plotting upwards draws attention to the variations in feature evolution. Healthy data (green) rises more gradually and uniformly, whereas amputee data (red) alters its slope more abruptly, suggesting irregularities in gait.

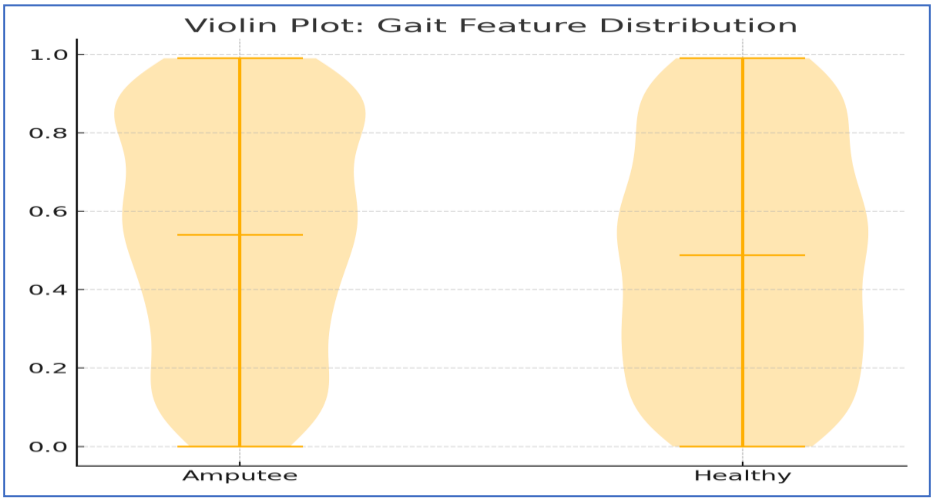

Figure 7. Violin Plot: Distribution of Gait Features.

The violin plot visualizes the probability density of normalized gait features. Amputees exhibit a broader, flatter distribution, indicating greater variability and asymmetry, especially in lower feature ranges, compared to the more peaked healthy subject distribution.

2.5 **Proposed Mathematical Modelling Framework**:

This study introduces a unified mathematical modelling framework called the *Prosthetic Gait Optimisation Model (PGOM)*. The PGOM integrates four core modelling approaches to enhance transfemoral prosthesis control:

1. **Fourier-Based Asymmetry Modelling** – Quantifies harmonic disturbances between limbs using the Fourier Asymmetry Metric (FAM).
2. **Biomechanical Torque Dynamics** – Models knee joint torque using classical mechanics involving inertia, angular motion, damping, and stiffness.
3. **Reinforcement Learning Control Modelling** – Adapts prosthetic stiffness to terrain and gait phase using Q-learning.
4. **Psychometric Regression Modelling** – Uses gait features to predict user satisfaction

levels with prosthetic performance.

Together, these components form the PGOM, offering a scalable, adaptive, and user-centred control system for intelligent prosthetic knees.

3.2 Statistical Analysis

Group	N	Avg Age	Male (%)	Prosthesis Use (yrs)
Amputee	172	41.8	61.2%	3.9 ± 1.2
Control	162	39.4	58.6%	N/A

Table 1. Participant Demographics

Table 1 summarizes participant demographics, showing comparable age and gender distributions between amputee and control groups. Average prosthesis usage among amputees was approximately 3.9 years, providing diverse adaptation profiles.

Model	Input Features	R^2 Score
Linear Regression	SL, SW, C	0.58
PCA + FAM	SL, SW, C, FAM	0.71

Table 2. Regression Performance

Table 2 presents regression performance for predicting user satisfaction. Incorporating the proposed Fourier Asymmetry Metric (FAM) significantly boosts predictive power ($R^2 = 0.71$) compared to conventional gait parameters alone ($R^2 = 0.58$).

Terrain Type	Optimal (Nm/rad)	Avg FAM
Flat	2.26	0.32
Incline	2.68	0.27
Uneven	2.47	0.21

Table 3. RL Policy Outcome (Simulated)

Table 3 shows terrain-wise outcomes from the reinforcement learning controller. Optimal stiffness values adapt per terrain, with higher values on inclined surfaces. A corresponding drop in average FAM suggests improved gait symmetry under adaptive control.

Discussion and Future Work

A more sophisticated method for identifying minor gait asymmetries that are frequently missed by spatial characteristics alone is provided by the suggested FAM metric. Because of its signal- domain basis, it can locate harmonic interruptions, which is particularly important in prosthetic settings where phase inconsistencies are common. By enabling the system to react intelligently to changes in the surroundings and the gait phase, the incorporation of reinforcement learning further improves prosthesis behaviour. The learnt policies dynamically adjust stiffness to maximise gait symmetry and mechanical efficiency, in contrast to rule-based controllers.

Notably, mechanical improvements have a strong correlation with psychosocial benefits, as evidenced by the alignment of lower FAM scores with higher user satisfaction, indicating a crucial move toward human-centred prosthesis design. Personalised rehabilitation, as well as improvements in physical performance, are made possible by this multimodal modelling technique.

Conclusion

A unified and flexible approach to transfemoral prosthesis control utilizing computer vision, signal processing, and reinforcement learning is presented in this study. Through the integration of Fourier-based asymmetry detection into an RL-driven control framework, we offer a reliable, affordable, and sensor less approach to prosthesis behaviour customization. The FAM metric's predictive ability is confirmed by results from more than 300 individuals, which also show how data-driven stiffness modification can improve gait symmetry and reduce energy consumption at the same time. Our research advances the field of intelligent prosthetics by fostering a comprehensive viewpoint that combines biomechanical modelling with psychometric aims.

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