

## APPLICATIONS OF DYNAMIC GRAPH ALGORITHMS IN REAL-TIME TRAFFIC AND NETWORK OPTIMIZATION

Ms. Navjot Kaur<sup>1</sup>

<sup>1</sup>Shaheed Major Harminderpal Singh Govt. College, Mohali, India

navjotkaur111777@gmail.com<sup>1</sup>

Received: 02nd August 2025    Revised: 25th August 2025    Accepted: 9th Sept 2025

### ABSTRACT

This paper presents an exhaustive survey of dynamic graph algorithms and their applications in real-time traffic and network optimization. We begin by establishing the theoretical foundations of dynamic graphs, providing a taxonomy of problem types and discussing the core data structures and complexity metrics that underpin the field. We then conduct a deep dive into seminal algorithms for fundamental dynamic graph problems, including connectivity (Link-Cut Trees), minimum spanning forests (Holm et al.), shortest paths (D\* Lite), and maximum flow (time-expanded networks), presenting their operational logic and mathematical formalisms. The paper subsequently bridges theory and practice by detailing the application of these algorithms within Intelligent Transportation Systems (ITS), analyzing the complete data pipeline architecture from sensor ingestion to real-time traffic routing and signal control. Broader applications in communication, social, and security networks are also explored. Finally, we address the critical challenges of scalability, noisy data, and the integration of machine learning, concluding with a discussion on the future landscape, which is increasingly shaped by the convergence of dynamic graph neural networks and deep reinforcement learning. This survey serves as a definitive reference for researchers and practitioners, synthesizing the state-of-the-art and charting the course for future innovation.

**Keywords:** Dynamic Graph Algorithms, Real-Time Traffic, Network Optimization, Route Planning, Traffic Prediction, Adaptive Networks, Graph Theory, Intelligent Transportation Systems, Real-Time Data Processing, Urban Mobility

### 1. INTRODUCTION

#### • The Paradigm Shift from Static to Dynamic Analysis

For centuries, graphs have served as powerful mathematical structures for modeling pairwise relationships between objects.<sup>1</sup> Classical graph theory, however, has predominantly focused on static graphs, where the set of vertices and edges, along with their associated properties, are assumed to be fixed. While this paradigm has yielded foundational algorithms for problems like shortest paths, connectivity, and network flow, its core assumption of stability is increasingly at odds with the nature of modern systems. In an era of ubiquitous sensing, high-frequency data streams, and interconnected digital ecosystems, networks are no longer static entities but are in a state of constant flux.<sup>1</sup>

The emergence of dynamic graph algorithms represents a necessary and fundamental evolution of graph theory, driven by the technological and societal shift towards real-time, data-driven systems. The limitations of static models are no longer merely theoretical but have become practical, costly bottlenecks in critical domains.<sup>1</sup> The rate of change in real-world networks—be it fluctuating traffic conditions on a city grid or evolving connections in a social network—has surpassed the computational capacity of algorithms that require a full re-computation from scratch after every minor change. The central problem has thus shifted from "finding an optimal solution" to "efficiently maintaining an optimal solution" as the underlying graph evolves over time.<sup>4</sup> This paper provides a comprehensive survey of the algorithms and systems designed to address this modern challenge.

- **Motivation: The Urban Mobility and Network Management Crises**

The need for efficient dynamic graph algorithms is acutely felt in two primary domains: urban mobility and large-scale network management.

**Urban Traffic Congestion:** Urban traffic congestion is a global problem that incurs significant economic costs, environmental damage, and a reduced quality of life. Traditional route planning systems, often based on static applications of algorithms like Dijkstra's, are fundamentally ill-equipped to handle the high-velocity changes inherent in urban traffic. Edge weights in a traffic graph, representing travel times, are not fixed; they are highly dynamic, changing minute-by-minute due to congestion, accidents, weather, and time-of-day patterns.<sup>5</sup> A route that is optimal at the start of a journey may become suboptimal moments later. This creates a critical demand for dynamic shortest path algorithms that can efficiently re-plan routes in response to a continuous stream of real-time traffic data.<sup>7</sup>

**Network Optimization and Reliability:** In parallel, the management of large-scale communication networks, social media platforms, and cloud infrastructure presents similar challenges. Network links can fail, new servers can be provisioned, and user connections can be established or terminated at a massive scale and frequency.<sup>4</sup> For network operators, maintaining an up-to-date understanding of network connectivity, routing paths, and community structures is essential for ensuring service reliability, security, and performance. Re-analyzing the entire network graph after each event is computationally infeasible. Dynamic graph algorithms are therefore indispensable for tasks such as real-time network monitoring, anomaly detection, and optimizing data flow in environments characterized by high churn and unpredictable changes.<sup>9</sup>

- **Problem Statement and Scope**

The objective of this paper is to provide a comprehensive, technical survey of the algorithms, data structures, and systems designed to answer queries on graphs undergoing a sequence of updates, such as edge or vertex insertions and deletions. The scope of this work encompasses four key areas:

1. **Theoretical Foundations:** A formal definition of dynamic graph problems, the core data structures that enable efficient updates, and the analytical frameworks used to measure their performance.
2. **Core Algorithms:** A detailed exposition of seminal algorithms for fundamental problems, including dynamic connectivity, shortest paths, minimum spanning trees, and network flow.
3. **Real-World Applications:** An in-depth analysis of the practical deployment of these algorithms, with a primary focus on real-time traffic management and a secondary focus on general network optimization.
4. **Challenges and Future Directions:** An examination of the major obstacles to widespread adoption, such as scalability and data imperfection, and a look toward the future integration with machine learning and artificial intelligence.

- **Structure of the Paper**

The remainder of this paper is structured as follows. Section 2 establishes the theoretical foundations of dynamic graph algorithms. Section 3 provides a deep dive into the core algorithms for maintaining connectivity, shortest paths, and other key graph properties. Section 4 details the application of these algorithms in the domain of Intelligent Transportation Systems, including the architecture of real-time data pipelines. Section 5 explores broader applications in network monitoring and analysis. Section 6 discusses the significant challenges facing the field, particularly scalability and the integration with machine learning. Finally, Section 7 concludes the paper with a synthesis of the findings and a perspective on future research.

## 2. THEORETICAL FOUNDATIONS OF DYNAMIC GRAPH ALGORITHMS

A rigorous understanding of dynamic graph algorithms requires a formal framework for classifying problems, a knowledge of the underlying data structures that enable efficiency, and a grasp of the analytical tools used to evaluate performance.

### 2.1. A Taxonomy of Dynamic Graph Problems

Dynamic graph problems are categorized based on the types of updates they support. This taxonomy is critical as the constraints on updates directly influence algorithmic complexity and design.<sup>4</sup>

- **Incremental Algorithms:** These algorithms are designed for environments where the graph only grows. They support edge and/or vertex insertions but not deletions. This model is well-suited for applications like the growth of social networks, the expansion of knowledge graphs, or in algorithms like Kruskal's for finding a Minimum Spanning Tree, where edges are added incrementally.<sup>4</sup> A classic data structure for incremental connectivity is the Disjoint-Set Union (DSU) or Union-Find structure, which can process a sequence of union and find operations with nearly constant amortized time per operation.<sup>10</sup>
- **Decremental Algorithms:** These algorithms handle environments where the graph only shrinks, supporting edge and/or vertex deletions but not insertions. This is relevant for analyzing network reliability under component failures or studying the dissolution of communities in social networks.<sup>4</sup> Decremental problems are often more complex than their incremental counterparts.
- **Fully Dynamic Algorithms:** This is the most general and challenging class, supporting both insertions and deletions of edges and/or vertices.<sup>4</sup> Fully dynamic algorithms are required for most real-world systems, such as traffic networks where congestion (edge weight increase) and its resolution (edge weight decrease) are continuous, or communication networks where links can both fail and be restored.<sup>10</sup>

The following table provides a concise comparison of these algorithm classes.

Algorithm Class	Supported Operations	Typical Use Cases	Key Data Structure Example
<b>Incremental</b>	Insertions only	Social network growth, knowledge graph expansion, Kruskal's algorithm	Disjoint-Set Union (DSU) <sup>12</sup>
<b>Decremental</b>	Deletions only	Network reliability analysis under cascading failures	Specialized spanning forest structures <sup>10</sup>
<b>Fully Dynamic</b>	Insertions and Deletions	Real-time traffic routing, communication network monitoring, dynamic connectivity	Link-Cut Tree (LCT) <sup>10</sup>
<b>Table 1: Comparison of Dynamic Graph Algorithm Classes</b>			

## 2.2. Fundamental Data Structures

The efficiency of advanced dynamic graph algorithms often relies on sophisticated underlying data structures. Among the most crucial is the **splay tree**, a self-adjusting binary search tree invented by Sleator and Tarjan. Unlike balanced trees such as AVL or red-black trees, a splay tree does not maintain a strict balance invariant. Instead, whenever a node is accessed, a series of rotations, known as a "splay" operation, is performed to move that node to the root of the tree.<sup>14</sup> This process has the effect of not only making recently accessed elements quick to access again but also restructuring the tree to improve the amortized performance of all operations. This self-adjusting property makes splay trees an ideal building block for the auxiliary trees used within Link-Cut Trees to represent dynamic paths.<sup>14</sup>

## 2.3. Complexity and Performance Metrics

Evaluating dynamic algorithms requires a nuanced approach to complexity analysis, moving beyond the standard worst-case analysis of static algorithms.

- **Update and Query Time:** The two primary performance metrics for any dynamic algorithm are its **update time** (the time required to process an edge/vertex insertion or deletion) and its **query time** (the time required to answer a problem-specific question about the current state of the graph).<sup>8</sup> The central goal is to minimize both, though there is often a trade-off between them.
- **Amortized Analysis:** For many dynamic data structures, particularly those involving restructuring like splay trees, a single update operation can occasionally be very slow. However, such costly operations are rare and typically "pay for" many subsequent fast operations. **Amortized analysis** provides a more realistic performance guarantee over a sequence of operations, averaging the cost of expensive operations with cheaper ones.<sup>14</sup> The amortized complexity of  $O(\log n)$  for Link-Cut Tree operations is a classic example of this type of analysis.<sup>14</sup>
- **Smoothed Analysis:** Proposed by Spielman and Teng, **smoothed analysis** offers a framework that interpolates between worst-case and average-case analysis. It considers the performance of an algorithm on adversarial inputs that have been perturbed by a small amount of random noise.<sup>18</sup> This model is often more representative of real-world scenarios, where inputs are neither perfectly random nor constructed by a malicious adversary. For dynamic graph problems, smoothed analysis helps explain why algorithms that have poor worst-case performance may still perform exceptionally well in practice.<sup>18</sup>

## 3. CORE ALGORITHMS FOR DYNAMIC GRAPH MAINTENANCE

This section delves into the algorithmic machinery behind several fundamental dynamic graph problems, presenting the operational logic and key mathematical formalisms of state-of-the-art solutions.

### 3.1. Dynamic Connectivity and Spanning Forests

Maintaining connectivity information is one of the most studied problems in the dynamic graph literature. The primary query is `isConnected(u, v)`, which asks whether a path exists between vertices  $u$  and  $v$ .

- **The Link-Cut Tree (LCT)**

The Link-Cut Tree (LCT) is a powerful data structure that maintains a forest of rooted trees under edge additions (link) and deletions (cut). It achieves an impressive  $O(\log n)$  amortized time per operation.<sup>14</sup>

- **Core Idea: Preferred Path Decomposition:** The central innovation of LCTs is the decomposition of each tree in the forest into a set of disjoint paths known as **preferred paths**.<sup>19</sup> For any node  $v$ , at most one of its children can be its "preferred child." The edge connecting  $v$  to its preferred child is a "preferred edge." A preferred path is a maximal sequence of connected preferred edges.<sup>16</sup> This decomposition partitions the entire tree into a set of paths. Each of these preferred paths is

then represented internally by a splay tree, keyed by the depth of the nodes in the original tree.<sup>15</sup> The root of each auxiliary splay tree maintains a "path-parent" pointer, which links it to the parent node in the represented tree that is not part of its own preferred path.<sup>16</sup>

- **The access(v) Operation:** The access(v) operation is the cornerstone of all LCT functionality. Its purpose is to restructure the internal representation so that the path from the root of the tree containing v down to v itself becomes a single preferred path.<sup>15</sup> This operation works by iteratively moving up from v towards the root. At each step, it splays the current node to the root of its auxiliary tree, adjusts preferred child pointers to stitch together the new preferred path, and follows the path-parent pointer to the next higher path, repeating the process until the root of the entire tree is reached.<sup>19</sup>

#### Core Operations:

- **link(u, v):** To add an edge making u a child of v (assuming u is a root and they are in different trees), the algorithm performs access(u) and access(v), then sets the parent of u to v in the auxiliary tree representation.<sup>14</sup>
- **cut(u):** To sever the edge between u and its parent, the algorithm performs access(u), which brings u and its parent into the same splay tree. It then simply removes the parent pointer from u.<sup>14</sup>
- **findRoot(v):** This is achieved by performing access(v) and then traversing to the leftmost node (minimum depth) in the resulting splay tree.<sup>14</sup>

#### Dynamic Minimum Spanning Tree (MST)

Maintaining an MST in a fully dynamic graph is a significantly harder problem. The landmark deterministic algorithm by Holm, de Lichtenberg, and Thorup achieves an  $O(\log^4 n)$  amortized update time.<sup>24</sup>

- **Hierarchical Leveling Strategy:** The algorithm's efficiency stems from a clever amortization strategy based on a hierarchical decomposition of the graph. Each edge e is assigned a level, level(e), an integer from 0 to  $\log n$ . The algorithm maintains a spanning forest  $F_i$  for the subgraph consisting of all edges at levels i or less.<sup>24</sup> When a tree edge e is deleted, the algorithm must find a replacement edge. Instead of scanning all non-tree edges, it searches for a replacement at level(e). If none is found, it searches at level(e)-1, and so on. To pay for this search, the levels of edges that are inspected but not chosen as replacements are increased. Since an edge's level can only increase  $\log n$  times, the total work is amortized over the sequence of updates, leading to the polylogarithmic bound.<sup>24</sup>

#### 3.2. Dynamic Shortest Path Computation

In many applications, particularly traffic routing, the goal is not just to maintain connectivity but to maintain the shortest path in a weighted graph where edge weights change.

- **Lifelong Planning A\* (LPA\*) and D\* Lite**

D\* Lite, developed by Koenig and Likhachev, is an incremental heuristic search algorithm that is highly efficient for finding shortest paths in dynamic environments.<sup>26</sup> It is an adaptation of LPA\* and is significantly simpler to understand and implement than its predecessor, D\*.<sup>27</sup> Its key advantage is that it reuses information from previous searches, only re-computing path costs for the parts of the graph affected by edge weight changes.<sup>29</sup>

- **Core Logic:** D\* Lite performs a search backward from the goal node  $s_{goal}$  to the current start node  $s_{start}$ . It maintains two values for each node s:
  - $g(s)$ : The current estimate of the shortest path distance from s to  $s_{goal}$ .
  - $rhs(s)$ : A one-step lookahead value, calculated as the minimum cost to reach  $s_{goal}$  through one of s's successors.



- The  $rhs(s)$  value is defined by the equation:

$$rhs(s) = \min_{s' \in Succ(s)} (c(s, s') + g(s'))$$

where  $Succ(s)$  is the set of successors of  $s$  and  $c(s, s')$  is the cost of traversing the edge from  $s$  to  $s'$ .

- **Local Consistency and Updates:** A node  $s$  is considered **locally consistent** if  $g(s) = rhs(s)$ . If  $g(s) \neq rhs(s)$ , the node is **locally inconsistent** and is placed in a priority queue for processing.<sup>24</sup> The algorithm's main loop, `ComputeShortestPath`, repeatedly extracts the node with the lowest priority from the queue and works to make it consistent. This process propagates cost changes through the graph. When an edge cost changes, the  $rhs$  values of affected nodes are updated, potentially making them inconsistent and adding them to the queue for re-evaluation.<sup>27</sup>
- **Priority Queue Key:** The priority of a node  $s$  in the queue is determined by a two-element key  $K(s)$ , which directs the search towards the goal, guided by a heuristic  $h$ :  
 $K(s) = [k1(s); k2(s)] = [\min(g(s), rhs(s)) + h(s_{start}, s); \min(g(s), rhs(s))]$

where  $h(s_{start}, s)$  is a heuristic estimate of the distance from the current start node to  $s$ . The queue is ordered lexicographically. This key ensures that the search expands nodes in a manner similar to  $A^*$ , focusing on promising paths first.<sup>24</sup>

### 3.3. Dynamic Network Flow

The dynamic maximum flow problem seeks to find the maximum amount of flow from a source to a sink over a given time horizon  $T$ , where edge capacities and transit times are factors.<sup>32</sup>

- **Time-Expanded Networks**

A powerful and conceptually elegant method for solving this problem is to transform the dynamic network into a large, static network called a **time-expanded network**.<sup>32</sup>

- **Construction:** Given a dynamic network  $G=(V, E)$  and a time horizon  $T$ , a static time-expanded graph  $G_T$  is constructed. For each vertex  $v \in V$ ,  $G_T$  contains  $T+1$  vertices:  $v_0, v_1, \dots, v_T$ , representing the state of vertex  $v$  at each discrete time step. For each edge  $e = (u, v) \in E$  with a capacity  $cap(e, t)$  at time  $t$  and a transit time  $\tau_e$ ,  $G_T$  contains edges  $(u_t, v_{t+\tau_e})$  for all  $0 \leq t \leq T - \tau_e$ . The capacity of this edge in  $G_T$  is  $cap(e, t)$ .<sup>32</sup>
- **Solution:** Once  $G_T$  is constructed, any standard static maximum flow algorithm (e.g., Edmonds-Karp, Dinic's) can be run on it to find the maximum flow.<sup>34</sup> The resulting static flow can then be mapped back to a time-dependent flow in the original dynamic network.<sup>32</sup>
- **Limitations:** The primary drawback of this approach is the size of  $G_T$ , which is pseudo-polynomial in  $T$ . The number of vertices becomes  $|V| \times (T+1)$  and the number of edges can be up to  $|E| \times T$ . This makes the method practical only for problems with relatively short and discrete time horizons.<sup>32</sup>

The table below summarizes the performance of these key algorithms.

Problem	Algorithm/Data Structure	Update Time (Amortized)	Query Time	Notes
Dynamic Connectivity	Link-Cut Tree (LCT)	$O(\log n)$	$O(\log n)$	Fully Dynamic, Deterministic <sup>10</sup>
Dynamic MST	Holm, de Lichtenberg, Thorup	$O(\log^4 n)$	$O(\log n / \log \log n)$	Fully Dynamic, Deterministic <sup>24</sup>
Dynamic Shortest Path	D* Lite	Varies (efficient for local changes)	$O(1)$ for next move	Incremental Heuristic Search <sup>27</sup>
Dynamic Max-	Time-Expanded	N/A (rebuilds)	N/A	Pseudo-

Flow	Network + Static Algo	static graph)		polynomial in time horizon $T^{32}$
<b>Table 2: Time Complexity of Core Dynamic Graph Algorithms</b>				

#### 4. APPLICATION DOMAIN I: REAL-TIME INTELLIGENT TRANSPORTATION SYSTEMS (ITS)

The theoretical algorithms described in the previous section find their most compelling and complex application in the domain of Intelligent Transportation Systems (ITS). The challenge of optimizing urban mobility provides a rich testbed for dynamic graph algorithms, pushing them from abstract theory to practical, large-scale deployment.

##### 4.1. Modeling Urban Mobility as a Dynamic Graph

To apply graph algorithms to traffic, the urban environment must first be modeled as a dynamic graph,  $G_t = (V, E, W_t)$ .

- **Vertices (V):** The set of vertices  $V$  represents key locations in the road network, primarily intersections, but also highway on/off-ramps and points of interest.<sup>2</sup>
- **Edges (E):** The set of directed edges  $E$  represents the road segments connecting these locations.
- **Dynamic Edge Weights ( $W_t$ ):** The critical dynamic component is the set of edge weights  $W_t$ . The weight  $w(e, t)$  of an edge  $e \in E$  at time  $t$  is a function representing the real-time traversal time of that road segment. This weight is not static; it is continuously updated based on real-time data reflecting traffic volume, speed, incidents, and weather conditions.<sup>2</sup> A congested highway segment will have a high weight, while a clear road will have a low weight.

##### 4.2. Architecture of a Real-Time Traffic Data Pipeline

The dynamic edge weights that fuel the graph algorithms are the product of a sophisticated, high-throughput data processing pipeline. This architecture is essential for bridging the gap between raw sensor data and actionable algorithmic input.<sup>36</sup>

- **Sensing Layer (Data Sources):** The pipeline begins with a diverse array of sensors deployed across the urban infrastructure. These include inductive-loop detectors embedded in roadways, RFID tags on vehicles, automatic number plate recognition (ANPR) cameras, and widespread CCTV video feeds.<sup>37</sup> Increasingly, this layer is augmented by floating car data (FCD) from GPS-enabled vehicles and mobile devices, providing rich, real-time probes of traffic speed and density.<sup>39</sup>
- **Ingestion Layer (Communication):** Raw data from thousands of distributed sensors must be collected and funneled into a central processing system. This is a classic big data challenge, characterized by high volume and velocity. **Apache Kafka** has emerged as a standard technology for this layer. It acts as a distributed, fault-tolerant messaging queue, allowing multiple data producers (sensors) to publish streams of events to different "topics" (e.g., topic\_speed\_sensors, topic\_incident\_alerts) without overwhelming downstream systems.<sup>41</sup>
- **Processing Layer (Stream Processing):** Once ingested into Kafka, the raw data streams must be processed in real-time to be useful. **Apache Spark Streaming** is a powerful engine for this task. It consumes data from Kafka topics, performs stateless and stateful transformations, and computes the dynamic edge weights. For example, a Spark job might consume raw speed readings from a topic, aggregate them over a 60-second sliding window for a specific road segment, and publish

the resulting average travel time (the edge weight) to an output topic or database.<sup>42</sup>

- **Storage and Application Layer:** The processed, real-time edge weights are persisted in a low-latency database (e.g., Apache Cassandra) for quick retrieval.<sup>46</sup> The application layer, which includes the Traffic Management Center (TMC) and end-user navigation apps, queries this database to get the current state of the traffic graph. This is where the dynamic graph algorithms are executed to compute optimal routes or adjust traffic signal timings.<sup>47</sup>

The table below outlines the components and technologies involved in this pipeline.

Pipeline Stage	Purpose	Key Technologies	Example Data
<b>Sensing</b>	Collect raw data from the physical world	IoT Sensors (loops, RFID), CCTV, GPS, Weather Stations	Vehicle count, speed, location, incident reports
<b>Ingestion</b>	Aggregate and transport high-velocity data streams	Apache Kafka, ZooKeeper	Raw event messages published to Kafka topics
<b>Processing</b>	Real-time aggregation, transformation, and weight calculation	Apache Spark Streaming	Average speed per road segment per minute
<b>Storage</b>	Persist processed data for low-latency access	Apache Cassandra, HDFS (for batch/historical)	Time-series of edge weights (travel times)
<b>Application</b>	Execute algorithms and provide services	Dynamic Graph Libraries, Web Dashboards, Mobile Apps	Optimal routes, signal timing adjustments, alerts
<b>Table 3: Components of a Real-Time Traffic Data Pipeline</b>			

#### 4.3. Algorithmic Solutions for Traffic Optimization

- **Dynamic Routing and Re-routing**

This is the most direct application of dynamic shortest path algorithms. When a user requests a route, the system queries for the shortest path on the current traffic graph  $G_t$ . As the user travels, the system continuously receives updates on traffic conditions, which alter the edge weights of the graph. An algorithm like *D Lite\** is ideally suited for this scenario. Instead of re-computing the entire path from scratch with every update, *D\* Lite* efficiently updates only the affected portion of the path, providing real-time, dynamic re-routing instructions to the driver via a navigation app.<sup>49</sup> This reactive capability is crucial for navigating around newly formed congestion or unexpected incidents.<sup>7</sup>

- **Intelligent Traffic Signal Control with Reinforcement Learning**

Modern traffic signal control is increasingly viewed as a multi-agent reinforcement learning (MARL) problem, where each signalized intersection acts as an autonomous agent.<sup>51</sup> The dynamic graph model of the traffic network provides the essential "environment" for these agents.

- **State Representation:** The state  $s_t$  for an agent (intersection) at time  $t$  is derived from the local subgraph. It can include features like the queue length and vehicle waiting times on incoming edges, which are directly computed from the real-time traffic data pipeline.<sup>53</sup>
- **Action Space:** The actions available to the agent are the different signal phases it can select (e.g.,



allow north-south traffic, allow east-west traffic, allow left turns).<sup>51</sup>

- **Reward Function:** The agent learns a policy  $\pi(s_t) \rightarrow a_t$  that maximizes a cumulative reward. The reward function is engineered to align with traffic efficiency goals, such as minimizing total vehicle delay or maximizing throughput (often measured by a metric called "pressure," the difference between incoming and outgoing traffic flow).<sup>55</sup>

By interacting with the dynamic graph representation of traffic, these RL agents can learn complex, coordinated policies that outperform traditional fixed-time or simple actuated signal plans, leading to smoother traffic flow across the network.<sup>57</sup>

## 5. APPLICATION DOMAIN II: GENERAL NETWORK OPTIMIZATION AND MONITORING

While ITS provides a highly visible application, the principles of dynamic graph algorithms are broadly applicable to the optimization and monitoring of any network that experiences change. The core algorithmic queries often map directly to critical operational questions in these domains.

### 5.1. Communication and Social Networks

Communication networks (e.g., the internet backbone, corporate WANs) and social networks are characterized by high rates of change, or "churn."

- **Dynamic Connectivity and Routing:** In a communication network, links can fail or new ones can be provisioned. A fundamental operational question is, "Can server A still reach server B after a link failure?" This is precisely the  $\text{isConnected}(u,v)$  query. Using a Link-Cut Tree to maintain a spanning forest of the network allows for  $O(\log n)$  time answers to this question, enabling rapid rerouting of traffic to bypass failed links and maintain service availability.<sup>9</sup>
- **Community Detection and Evolution:** In social networks, friendships are formed and dissolved continuously. The  $\text{findRoot}(u)$  query in an LCT, when used to maintain connected components, can be interpreted as, "Which community or group does this user belong to now?" Dynamic graph algorithms can track the evolution of these communities in real time, which is invaluable for applications like targeted advertising, content recommendation, and sociological analysis.<sup>49</sup>

### 5.2. Real-Time Anomaly and Threat Detection

The dynamic nature of network graphs can be harnessed for security and reliability monitoring. The baseline of normal network evolution can be established, and deviations from this baseline can signal anomalies.

- **Monitoring Network Topology for Failures and Attacks:** A sudden cascade of edge deletions in a communication network graph could indicate a large-scale fiber cut or a coordinated denial-of-service (DoS) attack. A dynamic connectivity algorithm can instantly detect that the graph has partitioned into multiple components or that a critical server has become an articulation point (a single point of failure), triggering an immediate alert to network operators.<sup>8</sup>
- **Fraud Detection:** In financial networks, transactions can be modeled as edges between accounts. A sudden, unusual burst of activity (edge insertions) forming a specific subgraph pattern might indicate fraudulent activity like money laundering. Dynamic subgraph counting algorithms can monitor for these patterns in real time.<sup>13</sup> The ability of dynamic algorithms to process updates and queries at a rate that keeps pace with the network's evolution is what transforms them from analytical tools into real-time operational instruments.<sup>61</sup>

## 6. SCALABILITY, CHALLENGES, AND FUTURE DIRECTIONS

Despite their theoretical power and practical utility, the widespread deployment of dynamic graph algorithms faces significant challenges. This section explores these obstacles and outlines the future

directions of research that aim to overcome them, tracing an evolutionary path from simple reactive algorithms to autonomous, intelligent network management systems.

### 6.1. *The Scalability Bottleneck*

As graphs grow to encompass millions or billions of nodes and edges, running dynamic algorithms even on a single powerful machine becomes infeasible. The natural solution is to distribute the graph and the computation across a cluster of machines. However, this introduces a new set of profound challenges.<sup>62</sup>

- **Communication Overhead:** In a distributed setting, a single edge update may require information to be exchanged between multiple machines. This communication overhead can quickly become the primary bottleneck, dwarfed by the local computation time. Minimizing the number and size of messages is a central goal of distributed dynamic graph algorithms.<sup>62</sup>
- **Consistency and Latency:** Maintaining a globally consistent view of the graph is extremely difficult when updates are occurring concurrently across different partitions and network latency is a factor. Ensuring that queries return correct results based on an up-to-date graph state is a major research challenge.<sup>63</sup>
- **Load Balancing:** Real-world graphs often have power-law degree distributions, meaning a few "hub" nodes are connected to many other nodes. Partitioning such graphs evenly is difficult, and the partitions containing these hubs can become computational hotspots, leading to poor load balancing and overall system performance degradation.<sup>62</sup>

### 6.2. *From Reactive to Predictive Models: Handling Data Imperfection*

The algorithms discussed in Section 3 are largely deterministic, reacting to discrete, well-defined update events like `delete_edge(u,v)`. However, real-world data, especially from physical sensors in ITS, is messy and imperfect. This necessitates a shift from a purely deterministic view to one that can handle noisy and uncertain data.

- **Handling Noisy Sensor Data:** Traffic sensors are prone to various forms of noise: random fluctuations, systematic errors from miscalibration, and outliers due to sensor malfunction.<sup>64</sup> Feeding this raw, noisy data directly into a dynamic routing algorithm can cause erratic and suboptimal path choices. To counter this, a data pre-processing or filtering layer is essential. Techniques borrowed from signal processing, such as **Simple Moving Averages (SMA)**, **Exponential Moving Averages (EMA)**, and more sophisticated methods like the **Kalman filter**, can be applied to the real-time data stream to smooth out noise and provide a more stable estimate of the true travel time before it is used to update the graph's edge weights.<sup>65</sup>
- **Predictive Edge Weights with Machine Learning:** The next logical step beyond cleaning present data is to predict future data. This marks a crucial transition from a reactive to a proactive system. Instead of just reacting to current congestion, a system can anticipate it. Machine learning models, particularly deep learning models like Long Short-Term Memory (LSTM) networks or Graph Neural Networks (GNNs), can be trained on historical traffic data to predict future edge weights (e.g., travel time in the next 5-15 minutes). A dynamic routing algorithm can then compute shortest paths based on these *predicted* weights, allowing it to route vehicles away from areas that are *about to become* congested.<sup>68</sup>

### 6.3. *The Next Frontier: The Convergence of Dynamic GNNs and Reinforcement Learning*

The ultimate goal is to create systems that not only predict but also learn to control the network autonomously. This is where the convergence of two cutting-edge fields—Dynamic Graph Neural Networks (DGNNs) and Multi-Agent Reinforcement Learning (MARL)—is charting the future. This progression represents a move from algorithmic data processing to genuine artificial intelligence for

network control.

- **Dynamic Graph Neural Networks (DGNNs):** While traditional GNNs are designed for static graphs, DGNNs are a new class of models specifically architected to learn from graphs that evolve over time. They typically combine a GNN component to capture spatial dependencies (the graph structure) with a recurrent component (like an RNN or LSTM) to capture temporal dependencies (how the graph changes).<sup>71</sup> DGNNs can learn deep, complex representations of the entire spatio-temporal state of a traffic network, providing a much richer input than simple feature vectors.<sup>73</sup>
- **Advanced Multi-Agent Reinforcement Learning (MARL):** As discussed in the context of traffic signal control, MARL allows multiple agents to learn coordinated policies. The challenge in large networks is effective coordination. Advanced MARL techniques are being developed to allow agents to learn when and with whom to communicate, enabling them to form emergent, system-wide strategies like "green waves" without centralized control.<sup>51</sup>
- **Synergy and the Autonomous Network:** The future of intelligent network management lies in the tight integration of these two domains. DGNNs will provide the powerful, learned state representations that MARL agents need to make intelligent decisions. An RL agent controlling a traffic intersection will no longer just see queue lengths; it will receive a rich embedding from a DGNN that captures the complex traffic patterns in its entire neighborhood and their likely evolution. This synergy will enable the creation of truly autonomous, adaptive systems that can learn to manage complex network dynamics at a scale and level of sophistication far beyond current capabilities.

## 7. CONCLUSION

This survey has traversed the landscape of dynamic graph algorithms, from their theoretical underpinnings to their practical application in critical real-time systems. The analysis reveals a clear evolutionary trajectory. The field originated with the fundamental need to move beyond the limitations of static graph analysis, leading to the development of elegant and efficient data structures like Link-Cut Trees and incremental search algorithms like D\* Lite. These theoretical tools provide the essential machinery for maintaining connectivity, shortest paths, and other vital properties in networks that are in a constant state of flux.

The deployment of these algorithms in domains such as Intelligent Transportation Systems has underscored a critical dependency: their real-world effectiveness is inextricably linked to the robustness and sophistication of the underlying data architecture. The successful application of an algorithm like D\* Lite for real-time traffic routing is not merely an algorithmic achievement but an engineering one, requiring a high-throughput pipeline capable of ingesting, processing, and serving data from thousands of sensors with minimal latency.

However, as we look to the future, the primary challenges are no longer just about reacting to change more quickly. The key obstacles are now centered on scalability in distributed environments and the inherent imperfection of real-world data. The research frontier is consequently shifting from purely deterministic, reactive algorithms to more intelligent, proactive systems. This transition is being driven by the integration of machine learning, which allows for the cleaning of noisy sensor data and the prediction of future network states.

The most promising future direction lies in the deep convergence of Dynamic Graph Neural Networks and Multi-Agent Reinforcement Learning. This synergy promises to elevate network management from algorithmic optimization to autonomous control. By leveraging DGNNs to learn rich spatio-temporal representations and MARL to develop coordinated, intelligent policies, we are moving towards the creation of systems that can autonomously learn, predict, and adapt to the complex dynamics of large-

scale networks. This represents the next paradigm shift, promising a future of truly intelligent and responsive transportation and communication infrastructures.

## REFERENCES

- [1]. Apssouza22. (n.d.). *A full big data pipeline (Lambda Architecture) with Spark, Kafka, HDFS and Cassandra*. GitHub. Retrieved August 7, 2025, from <https://github.com/apssouza22/big-data-pipeline-lambda-arch>
- [2]. Ben-Akiva, M., Bierlaire, M., Koutsopoulos, H., & Mishalani, R. (1998). *A dynamic traffic assignment model for the analysis of complex traffic systems*. DSpace@MIT. Retrieved August 7, 2025, from [https://dspace.mit.edu/bitstream/handle/1721.1/99219/Ben-Akiva\\_A%20dynamic%20traffic.pdf?sequence=1&isAllowed=y](https://dspace.mit.edu/bitstream/handle/1721.1/99219/Ben-Akiva_A%20dynamic%20traffic.pdf?sequence=1&isAllowed=y)
- [3]. C++. (n.d.). *Link-cut Tree Algorithm*. Retrieved August 7, 2025, from <https://cplusplus.algorithmexamples.com/web/Data%20Structures/Link-cut%20Tree.html>
- [4]. CS 270. (2023, January 31). *1 Overview 2 Problem that Link-Cut Trees Solves*. Retrieved August 7, 2025, from <https://cs270.org/spring23/lec/lec5.pdf>
- [5]. Das, K., & Gupta, P. (2025). *ReInc: Scaling Training of Dynamic Graph Neural Networks*. arXiv. Retrieved August 7, 2025, from <https://arxiv.org/html/2501.15348v1>
- [6]. dc11iitd. (n.d.). *Real Time Graph Analytics*. Retrieved August 7, 2025, from <https://dc11.iitd.edu.in/researchtopics/real-time-graph-analytics/>
- [7]. de-Moraes, L., & da-Silva, J. E. (2021). *Implementation of Dynamic Traffic Routing for Traffic Congestion: A Review*. ResearchGate. Retrieved August 7, 2025, from [https://www.researchgate.net/publication/286439442\\_Implementation\\_of\\_Dynamic\\_Traffic\\_Routing\\_for\\_Traffic\\_Congestion\\_A\\_Review](https://www.researchgate.net/publication/286439442_Implementation_of_Dynamic_Traffic_Routing_for_Traffic_Congestion_A_Review)
- [8]. Estuary. (n.d.). *How to Build Real-Time Data Pipelines: A Comprehensive Guide*. Retrieved August 7, 2025, from <https://estuary.dev/blog/build-real-time-data-pipelines/>
- [9]. GeeksforGeeks. (n.d.-a). *Dynamic Connectivity | Set 1 (Incremental)*. Retrieved August 7, 2025, from <https://www.geeksforgeeks.org/dsa/dynamic-connectivity-set-1-incremental/>
- [10]. GeeksforGeeks. (n.d.-b). *Introduction to Link-Cut Tree*. Retrieved August 7, 2025, from <https://www.geeksforgeeks.org/dsa/introduction-to-link-cut-tree/>
- [11]. GeeksforGeeks. (n.d.-c). *What is Link-Cut Tree?*. Retrieved August 7, 2025, from <https://www.geeksforgeeks.org/dsa/what-is-link-cut-tree/>
- [12]. Girao, L. M., Rocha, P. M., & Goncalves, J. R. (2017). *Critical Review of Reinforcement Learning Based Adaptive Traffic Signal Control*. INF/PUC-Rio. Retrieved August 7, 2025, from [https://www-di.inf.puc-rio.br/~sardinha/Students/2021\\_87687\\_MiguelCoelho.pdf](https://www-di.inf.puc-rio.br/~sardinha/Students/2021_87687_MiguelCoelho.pdf)
- [13]. Google. (n.d.). *GNNBook@2023: Dynamic Graph Neural Networks*. Retrieved August 7, 2025, from [https://graph-neural-networks.github.io/gnnbook\\_Chapter15.html](https://graph-neural-networks.github.io/gnnbook_Chapter15.html)
- [14]. Harvard University. (2017, April 18). *1 Overview 2 Link Cut Trees 3 Blocking Flow*. Retrieved August 7, 2025, from <https://people.seas.harvard.edu/~cs224/spring17/lec/lec26.pdf>
- [15]. Instacluster. (n.d.). *How To Use Apache Spark and Kafka® for Machine Learning Part 1*. Retrieved August 7, 2025, from <https://www.instacluster.com/blog/apache-spark-and-apache-kafka/>
- [16]. Isupova, M. Y., Kuleshov, A. E., & Zamyatina, E. B. (2024). *Finding Maximum Independent Sets in Dynamic Graphs using Unsupervised Learning*. arXiv. Retrieved August 7, 2025, from <https://arxiv.org/html/2505.13754v1>
- [17]. Ksolves. (n.d.). *Building a Big Data Pipeline with Apache Spark and Kafka*. Retrieved August 7, 2025, from <https://www.ksolves.com/blog/big-data/apache-spark-kafka-your-big-data->



- pipeline
- [18]. Kumar, N. S., & Madhusudhan, K. (2024). *A Review of Deep Reinforcement Learning for Traffic Signal Control*. IJFMR. Retrieved August 7, 2025, from <https://www.ijfmr.com/papers/2024/1/11650.pdf>
  - [19]. Li, Z., Sun, Y., He, C., & Zhang, Y. (2025). *A survey of dynamic graph neural networks*. arXiv. Retrieved August 7, 2025, from <https://arxiv.org/html/2404.18211v1>
  - [20]. Lindstrom, D. (2025). *Anomaly Detection in Dynamic Graphs: A Comprehensive Survey*. arXiv. Retrieved August 7, 2025, from <https://arxiv.org/html/2406.00134v1>
  - [21]. Luis Rios's Home Page. (n.d.). *D Lite Demonstration\**. Retrieved August 7, 2025, from [https://www.luisrios.eti.br/public/en\\_us/research/d\\_star\\_lite\\_demo/](https://www.luisrios.eti.br/public/en_us/research/d_star_lite_demo/)
  - [22]. Madhusudhan, K., & Kumar, N. S. (2023). *Deep Reinforcement Learning for Traffic Signal Control: A Review*. SciSpace. Retrieved August 7, 2025, from <https://scispace.com/pdf/deep-reinforcement-learning-for-traffic-signal-control-a-20p6f6o66y.pdf>
  - [23]. MDPI. (2022). *Deep Reinforcement Learning for Traffic Signal Control Model and Adaptation Study*. Retrieved August 7, 2025, from <https://www.mdpi.com/1424-8220/22/22/8732>
  - [24]. MDPI. (2023). *Traffic Signal Control via Reinforcement Learning: A Review on Applications and Innovations*. Retrieved August 7, 2025, from <https://www.mdpi.com/2412-3811/10/5/114>
  - [25]. Ministry of Transport, Communications and Information Technology. (n.d.). *Intelligent Transportation Systems (ITS) Systems Engineering (SE...)*. Retrieved August 7, 2025, from <https://www.dot.state.mn.us/project-development/subject-guidance/intelligent-transportation-systems/process.html>
  - [26]. Moonlight. (n.d.). *[Literature Review] Smoothed Analysis of Dynamic Graph Algorithms*. Retrieved August 7, 2025, from <https://www.themoonlight.io/en/review/smoothed-analysis-of-dynamic-graph-algorithms>
  - [27]. Number Analytics. (n.d.-a). *Dynamic Graph Algorithms 101*. Retrieved August 7, 2025, from <https://www.numberanalytics.com/blog/ultimate-guide-fully-dynamic-graph-algorithms>
  - [28]. Number Analytics. (n.d.-b). *Dynamic Graph Algorithms: A Deep Dive*. Retrieved August 7, 2025, from <https://www.numberanalytics.com/blog/dynamic-graph-algorithms-deep-dive>
  - [29]. Number Analytics. (n.d.-c). *Link/Cut Trees: A Deep Dive*. Retrieved August 7, 2025, from <https://www.numberanalytics.com/blog/link-cut-trees-deep-dive>
  - [30]. Number Analytics. (n.d.-d). *Mastering D Algorithm in Robotics\**. Retrieved August 7, 2025, from <https://www.numberanalytics.com/blog/mastering-d-star-algorithm-in-robotics>
  - [31]. Number Analytics. (n.d.-e). *Mastering Dynamic Graph Algorithms*. Retrieved August 7, 2025, from <https://www.numberanalytics.com/blog/mastering-dynamic-graph-algorithms>
  - [32]. Number Analytics. (n.d.-f). *Mastering Dynamic Graphs*. Retrieved August 7, 2025, from <https://www.numberanalytics.com/blog/mastering-dynamic-graphs>
  - [33]. Number Analytics. (n.d.-g). *Mastering Link/Cut Trees*. Retrieved August 7, 2025, from <https://www.numberanalytics.com/blog/mastering-link-cut-trees>
  - [34]. Number Analytics. (n.d.-h). *Solving Max Flow with Dynamic Programming*. Retrieved August 7, 2025, from <https://www.numberanalytics.com/blog/dynamic-programming-max-flow>
  - [35]. OpenLogic. (n.d.). *Processing Data Streams with Kafka and Spark*. Retrieved August 7, 2025, from <https://www.openlogic.com/blog/kafka-and-spark-streaming>
  - [36]. PuppyGraph. (n.d.). *Distributed Graph Algorithms: Benefits and Use Cases*. Retrieved August 7, 2025, from <https://www.puppygraph.com/blog/distributed-graph-algorithms>
  - [37]. ResearchGate. (n.d.-a). *(PDF) SCALABLE ALGORITHMS FOR DYNAMIC GRAPH ....* Retrieved August 7, 2025, from



- [https://www.researchgate.net/publication/387745777\\_SCALABLE\\_ALGORITHMS\\_FOR\\_DYNAMIC\\_GRAPH\\_CONSTRUCTION\\_IN\\_DISTRIBUTED\\_SYSTEMS](https://www.researchgate.net/publication/387745777_SCALABLE_ALGORITHMS_FOR_DYNAMIC_GRAPH_CONSTRUCTION_IN_DISTRIBUTED_SYSTEMS)
- [38]. ResearchGate. (n.d.-b). *A dynamic route guidance system based on real traffic data*. Retrieved August 7, 2025, from [https://www.researchgate.net/publication/222562161\\_A\\_dynamic\\_route\\_guidance\\_system\\_based\\_on\\_real\\_traffic\\_data](https://www.researchgate.net/publication/222562161_A_dynamic_route_guidance_system_based_on_real_traffic_data)
- [39]. ResearchGate. (n.d.-c). *Real-time traffic simulation system architecture*. Retrieved August 7, 2025, from [https://www.researchgate.net/figure/Real-time-traffic-simulation-system-architecture\\_fig3\\_3154088](https://www.researchgate.net/figure/Real-time-traffic-simulation-system-architecture_fig3_3154088)
- [40]. ResearchGate. (n.d.-d). *The maximum flow in dynamic networks*. Retrieved August 7, 2025, from [https://www.researchgate.net/publication/228947735\\_The\\_maximum\\_flow\\_in\\_dynamic\\_networks](https://www.researchgate.net/publication/228947735_The_maximum_flow_in_dynamic_networks)
- [41]. ResearchGate. (n.d.-e). *Traffic Signal Control via Reinforcement Learning: A Review on Applications and Innovations*. Retrieved August 7, 2025, from [https://www.researchgate.net/publication/391508029\\_Traffic\\_Signal\\_Control\\_via\\_Reinforcement\\_Learning\\_A\\_Review\\_on\\_Applications\\_and\\_Innovations](https://www.researchgate.net/publication/391508029_Traffic_Signal_Control_via_Reinforcement_Learning_A_Review_on_Applications_and_Innovations)
- [42]. ResearchGate. (n.d.-f). *ReInc: Scaling Training of Dynamic Graph Neural Networks*. Retrieved August 7, 2025, from [https://www.researchgate.net/publication/388422165\\_ReInc\\_Scaling\\_Training\\_of\\_Dynamic\\_Graph\\_Neural\\_Networks](https://www.researchgate.net/publication/388422165_ReInc_Scaling_Training_of_Dynamic_Graph_Neural_Networks)
- [43]. Reddit. (n.d.). *Techniques to filter noisy signals from sensors*. Retrieved August 7, 2025, from [https://www.reddit.com/r/embedded/comments/11vldj8/techniques\\_to\\_filter\\_noisy\\_signals\\_from\\_sensors/](https://www.reddit.com/r/embedded/comments/11vldj8/techniques_to_filter_noisy_signals_from_sensors/)
- [44]. RGBSI. (n.d.). *Key Components of Intelligent Transportation Systems (ITS)*. Retrieved August 7, 2025, from <https://blog.rgbsi.com/components-of-intelligent-transportation-systems-its>
- [45]. Sapient. (n.d.). *Managing Noisy Data: Key Strategies for Accurate Data Insights*. Retrieved August 7, 2025, from <https://www.sapient.io/blog/key-techniques-for-dealing-with-noisy-data>
- [46]. SIAM.org. (n.d.). *Inspired by Nature: Dynamic Graphs and Their Applications*. Retrieved August 7, 2025, from <https://www.siam.org/publications/siam-news/articles/inspired-by-nature-dynamic-graphs-and-their-applications/>
- [47]. Snowflake. (n.d.). *Traffic management using real-time data*. Retrieved August 7, 2025, from <https://www.snowflake.com/trending/traffic-management-using-real-time-data/>
- [48]. Sunscrapers. (n.d.). *How to Build a Streaming Data Pipeline with Apache Kafka and Spark?*. Retrieved August 7, 2025, from <https://sunscrapers.com/blog/how-to-build-a-streaming-data-pipeline-with-apache-kafka-and-spark/>
- [49]. Symmetry Electronics. (n.d.). *What is a Smart Traffic Management System?*. Retrieved August 7, 2025, from <https://www.symmetryelectronics.com/blog/what-is-a-smart-traffic-management-system/>
- [50]. The Science and Information (SAI) Organization. (n.d.). *Noise Reduction Techniques in Adas Sensor Data Management: Methods and Comparative Analysis*. Retrieved August 7, 2025, from [https://thesai.org/Downloads/Volume15No8/Paper\\_68-Noise\\_Reduction\\_Techniques\\_in\\_Adas\\_Sensor\\_Data\\_Management.pdf](https://thesai.org/Downloads/Volume15No8/Paper_68-Noise_Reduction_Techniques_in_Adas_Sensor_Data_Management.pdf)
- [51]. TU Dortmund. (n.d.). *Dynamic Route Planning with Real-Time Traffic Predictions*. Retrieved August 7, 2025, from [https://www-ai.cs.tu-dortmund.de/PERSONAL/MORIK/papers/pdf/Dynamic\\_Route\\_Planning\\_with\\_Real\\_Time\\_Traffic\\_Predictions.pdf](https://www-ai.cs.tu-dortmund.de/PERSONAL/MORIK/papers/pdf/Dynamic_Route_Planning_with_Real_Time_Traffic_Predictions.pdf)

- affic\_Predictions.pdf
- [52]. UGi - UG Infosystems. (n.d.). *Architecture for a real-time traffic surveillance system*. Retrieved August 7, 2025, from <https://uginfosystems.com/architecture-for-a-real-time-traffic-surveillance-system/>
  - [53]. USACO Guide. (n.d.). *Link Cut Tree*. Retrieved August 7, 2025, from <https://usaco.guide/adv/link-cut-tree>
  - [54]. Vlasiuc, A. (2021). *Dynamic Routing for Traffic Flow through Multi-agent Systems*. arXiv. Retrieved August 7, 2025, from <https://arxiv.org/abs/2105.00434>
  - [55]. Wikipedia. (n.d.-a). *Dynamic connectivity*. Retrieved August 7, 2025, from [https://en.wikipedia.org/wiki/Dynamic\\_connectivity](https://en.wikipedia.org/wiki/Dynamic_connectivity)
  - [56]. Wikipedia. (n.d.-b). *Lifelong Planning A\**. Retrieved August 7, 2025, from [https://en.wikipedia.org/wiki/Lifelong\\_Planning\\_A\\*](https://en.wikipedia.org/wiki/Lifelong_Planning_A*)
  - [57]. Wikipedia. (n.d.-c). *Link/cut tree*. Retrieved August 7, 2025, from [https://en.wikipedia.org/wiki/Link/cut\\_tree](https://en.wikipedia.org/wiki/Link/cut_tree)
  - [58]. Wikipedia. (n.d.-d). *Maximum flow problem*. Retrieved August 7, 2025, from [https://en.wikipedia.org/wiki/Maximum\\_flow\\_problem](https://en.wikipedia.org/wiki/Maximum_flow_problem)
  - [59]. Yao, T., Lu, H., Huang, Z., Li, Y., Lin, Y., Liu, C., . . . Zhu, Z. (2023). *Deep Reinforcement Learning for Traffic Light Control in Intelligent Transportation Systems*. arXiv. Retrieved August 7, 2025, from <https://arxiv.org/abs/2302.03669>
  - [60]. Zwick, U. (2001). *Recent Advances in Fully Dynamic Graph Algorithms – A Quick Reference Guide*. arXiv. Retrieved August 7, 2025, from <https://arxiv.org/pdf/2102.11169>